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计算机与信息学院  
SCHOOL OF COMPUTER SCIENCE AND INFORMATION ENGINEERING  
人工智能学院  
SCHOOL OF ARTIFICIAL INTELLIGENCE

# 几何相位超表面近期研究进展

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合肥工业大学

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人工微结构光声调控物理与应用 学术研讨会



# 目录

## 1. 研究背景

2. 非线性几何相位超表面：广义非线性几何相位

3. 声学几何相位型超构表面

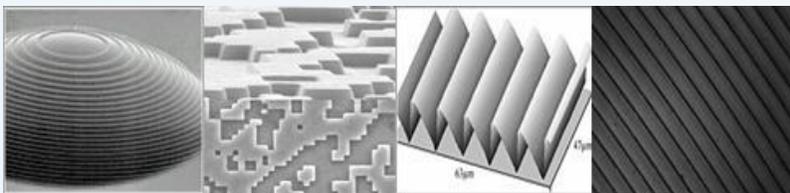
4. 总结



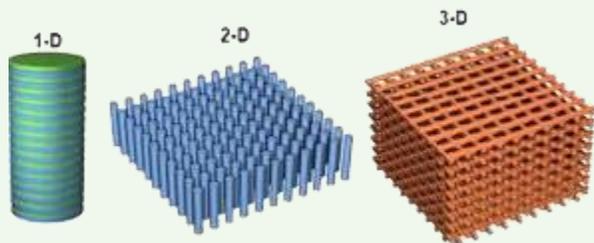
# 1. 研究背景



DOE



光子晶体

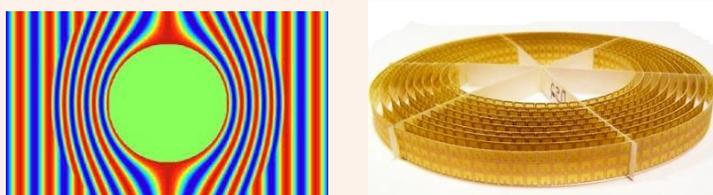


负折射



超材料

电磁隐身衣& 变换光学



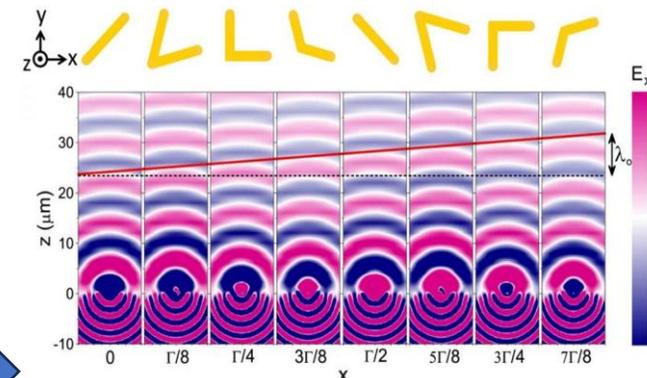
- 标量衍射理论
- 传播相位/光程

- 能带理论
- 周期体系的多重散射/非局域性

- 等效介质理论
- 超原子的局域共振
- 集体响应/非局域性

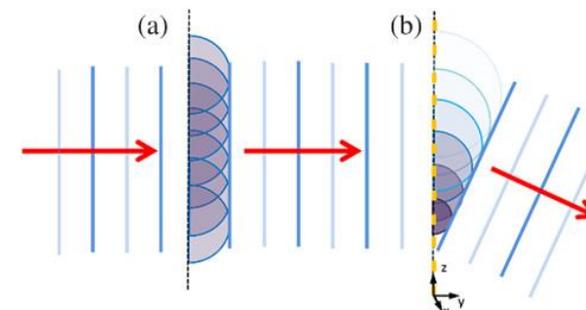
## 电磁波调控的核心: 相位

### 超表面: 2D 超构器件



Science, 2011, 334, 333.

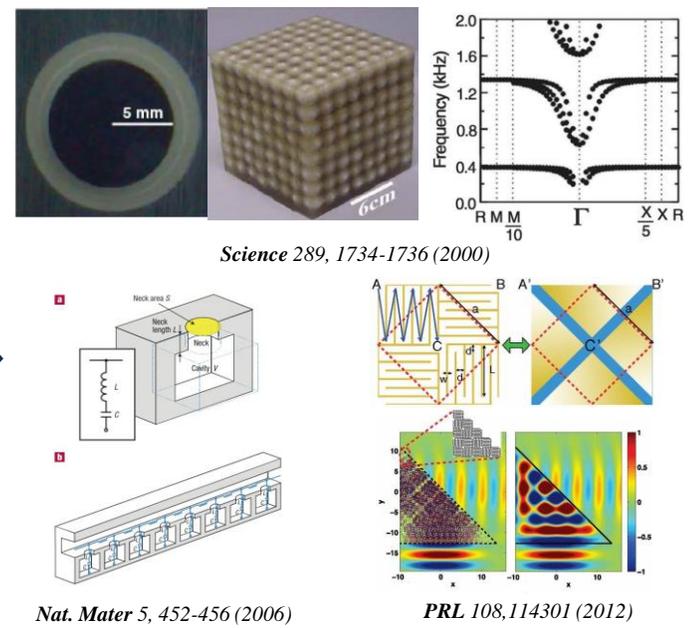
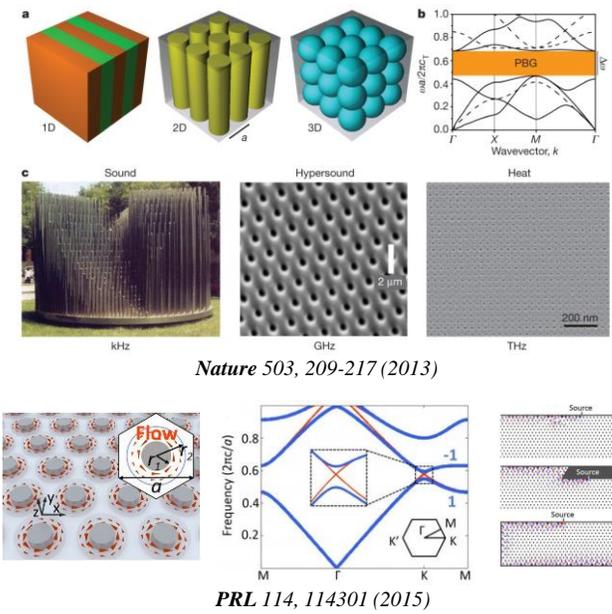
### 超原子散射特性独立设计



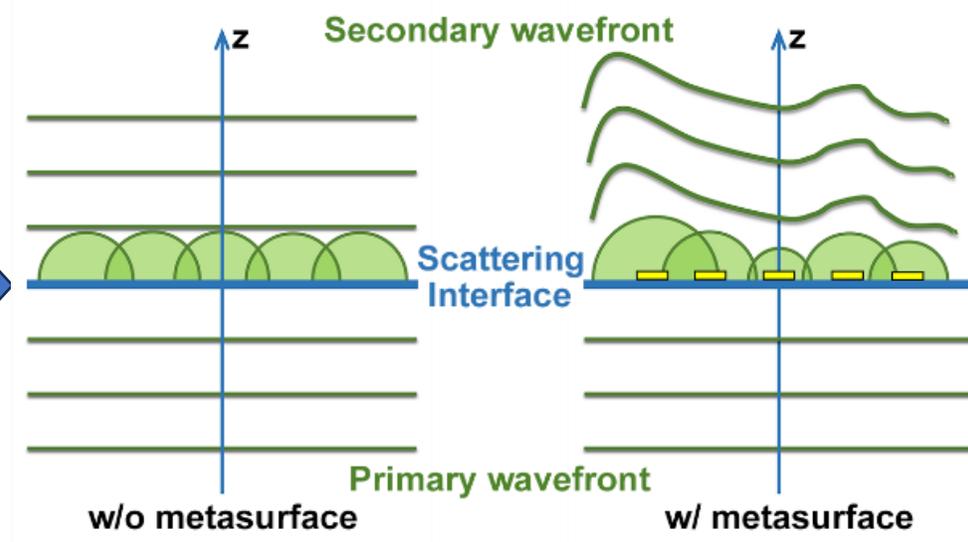
Optica, 2017,4(1),139-152.



# 1. 研究背景



## 声场调控核心：相位



声子晶体  
Bragg散射、能带理论

超构材料  
等效介质理论、局域共振

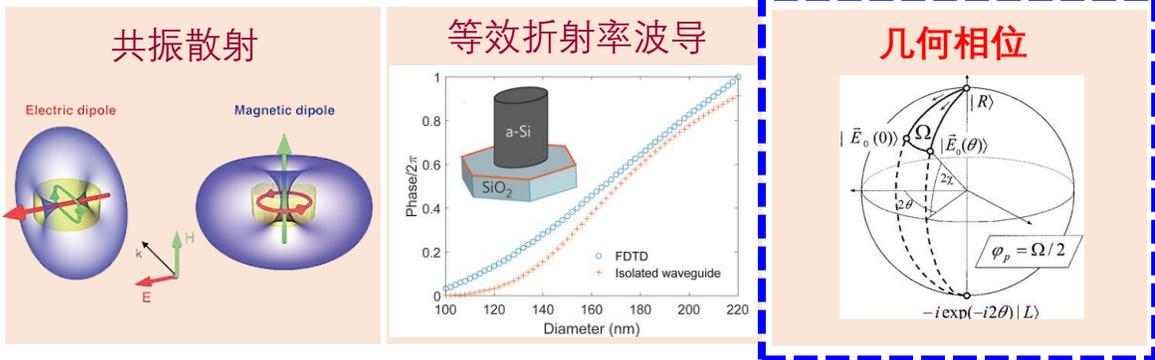
超构表面  
人造子波源、惠更斯原理



# 1. 研究背景

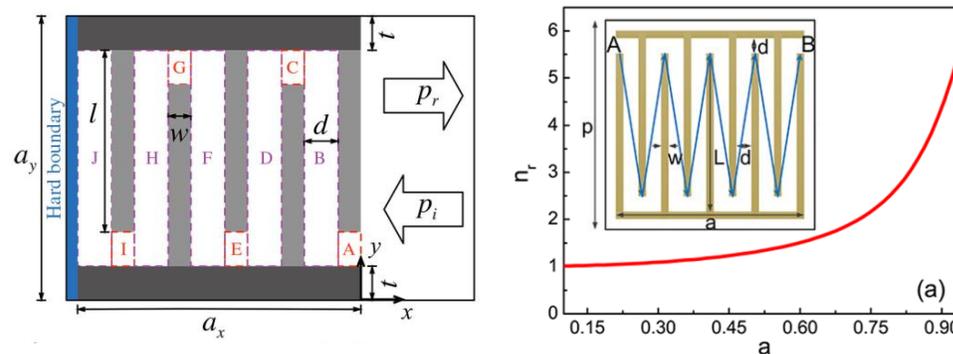


电磁超构表面相位调控基本方案:  
(1)共振相位; (2)传输相位; (3)几何相位.



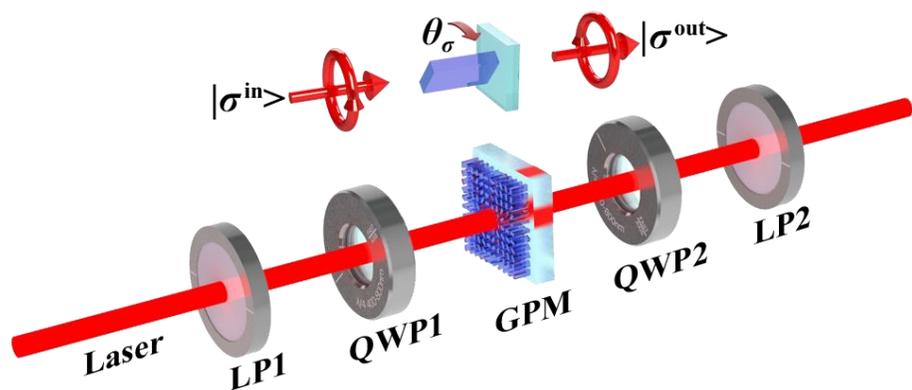
声学超构表面相位调控基本方案:  
(1)传输相位; (2)共振相位.

✓ 空间弯曲结构: 等效介质, 声阻抗/折射率



Phy. Rev. Appl., 2014,2,064002

Appl. Phys. Lett., 2012,101,233508



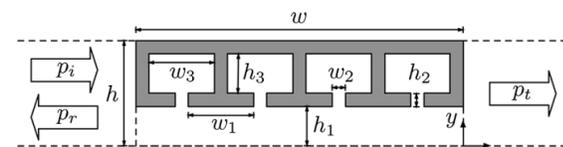
线性光学



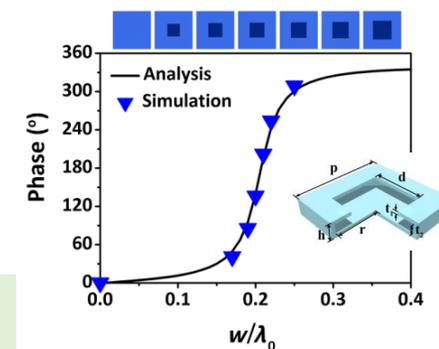
非线性光学

电磁几何相位:  $\exp[i(\sigma^{in} - \sigma^{out})\theta_\sigma]$

✓ 亥姆霍兹共振腔



Phy. Rev. Appl., 2015,4,024003



Phy. Rev. X, 2017,7,021034



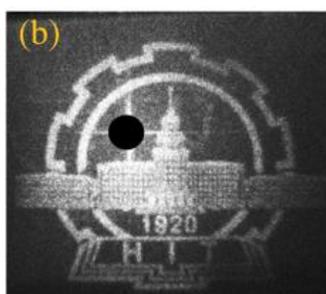
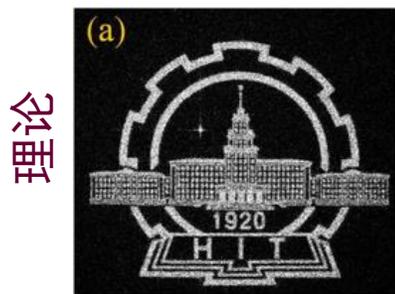
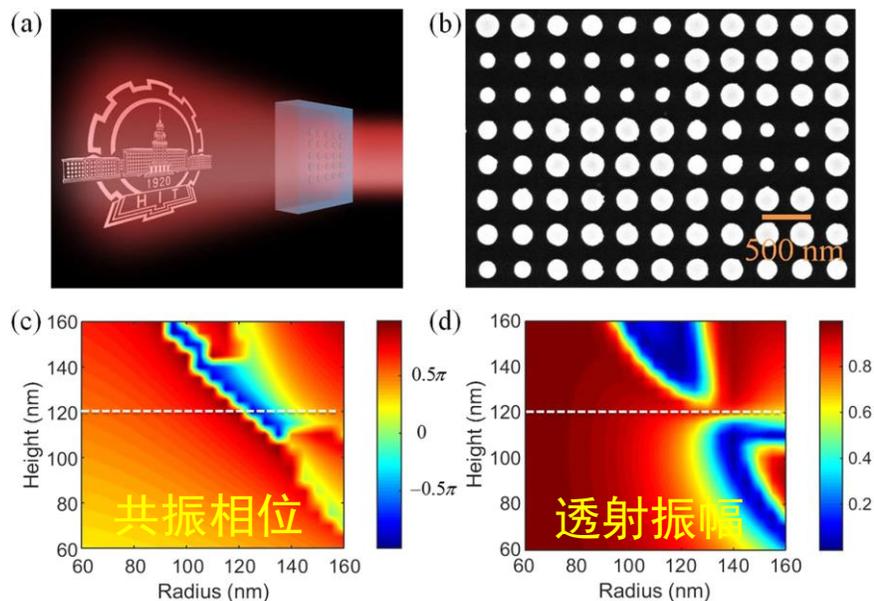
是否可以发展出  
声学几何相位的概念?



# 1. 研究背景



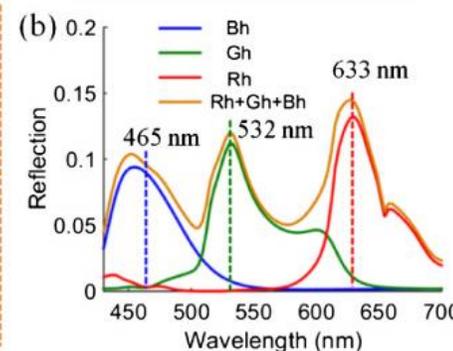
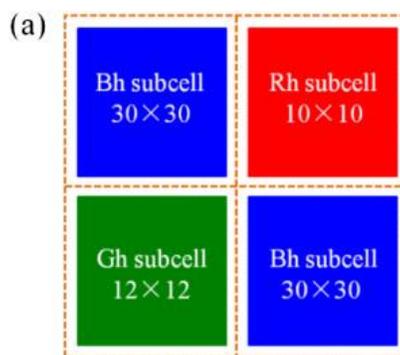
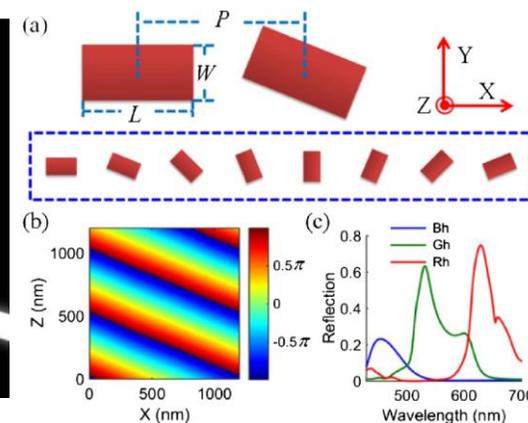
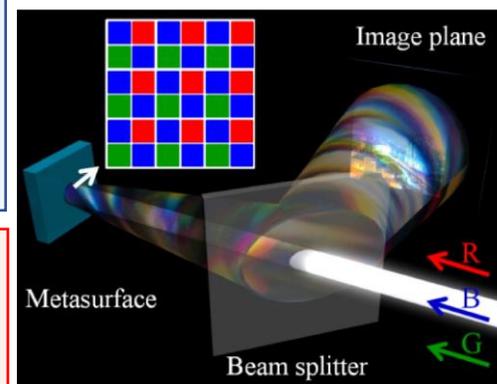
## ➤ 早期工作1: 基于介质圆盘的偏振不敏感近红外全息



W. Zhao, H. Jiang, **Bingyi Liu**, et al. *Sci. Rep.*, 6:30613 (2016)

## ➤ 早期工作2: 基于几何相位和共振响应设计的全彩色光信息处理

- 共振相位: 几何尺寸;
  - 频率响应: 几何尺寸
- 几何相位: 空间取向;
  - 频率响应: 几何尺寸



目标图像



重构图像



W. Zhao, **Bingyi Liu**, et al. *Opt. Lett.*, 4(1):147-150 (2016).



# 目录

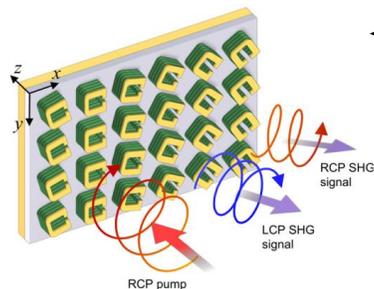
1. 研究背景
- 2. 非线性几何相位超表面：广义非线性几何相位**
3. 声学几何相位型超构表面
4. 总结



## 2. 非线性几何相位超表面



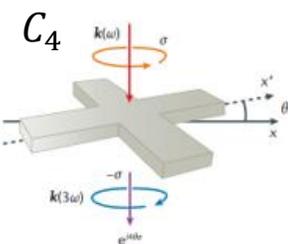
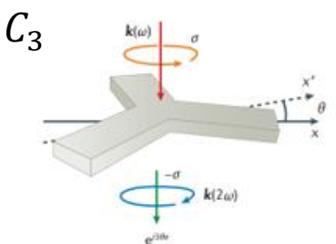
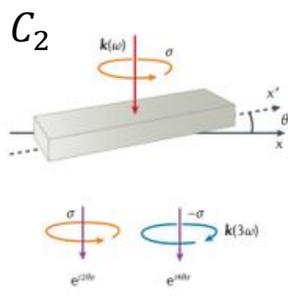
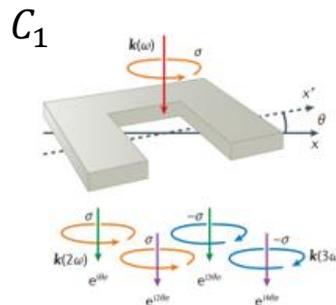
### 非线性几何相位超表面



自旋相关的非线性几何相位

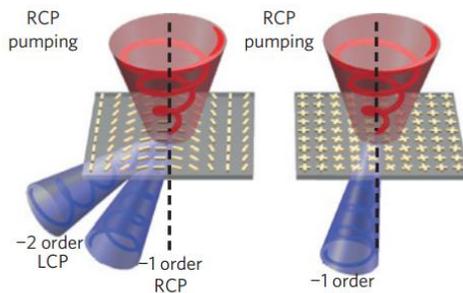
*Phys. Rev. Lett.*, 2015, 115, 207403.

选择定则：结构对称性决定谐波过程及几何相位

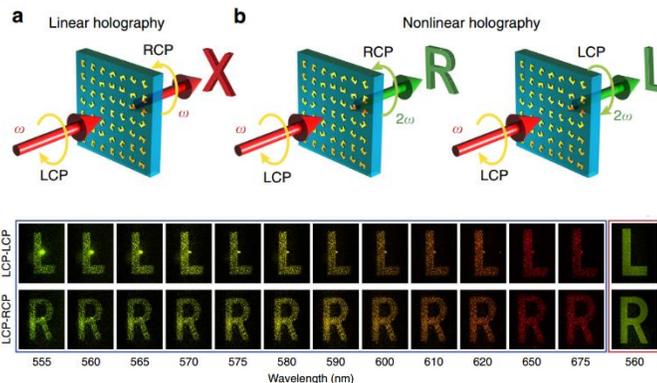


*Nat. Rev. Mat.*, 2017, 2: 17010.

### 金属型非线性几何相位超表面

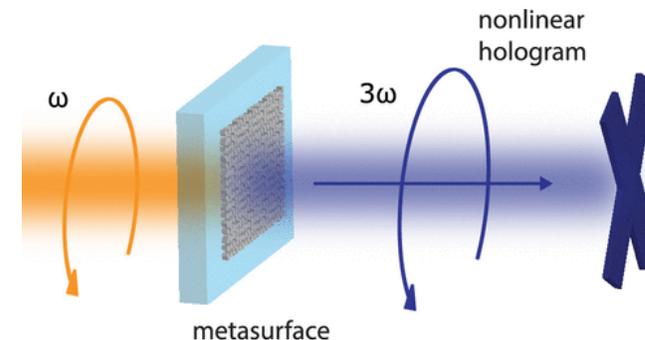


*Nat. Mater.*, 2015, 14, 607.

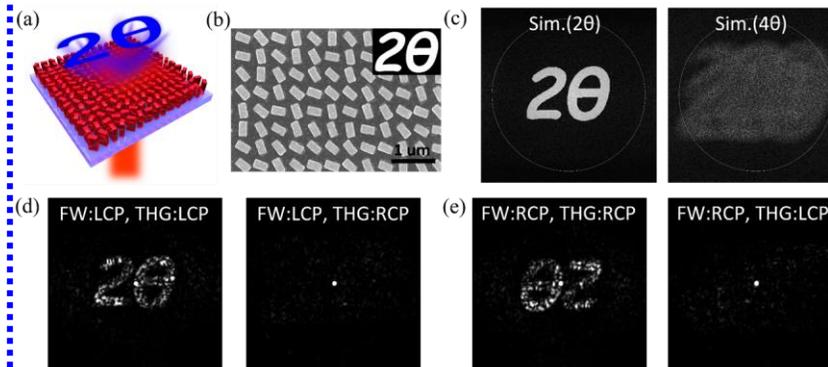


*Nat. Commun.*, 2016, 7:11930.

### 介质型非线性几何相位超表面



*Nano Lett.*, 2020, 19(9), 6585-6591.



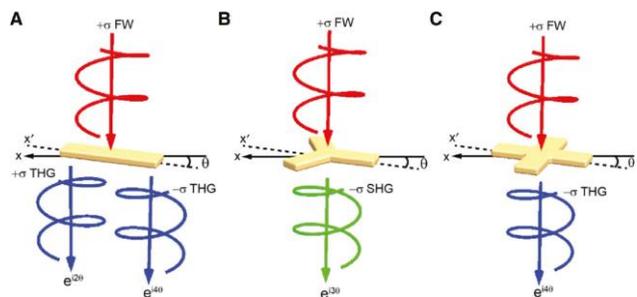
*Bingyi Liu, et al. Adv. Opt. Mat.*, 2020, 8(9), 1902050.



## 2. 非线性几何相位超表面



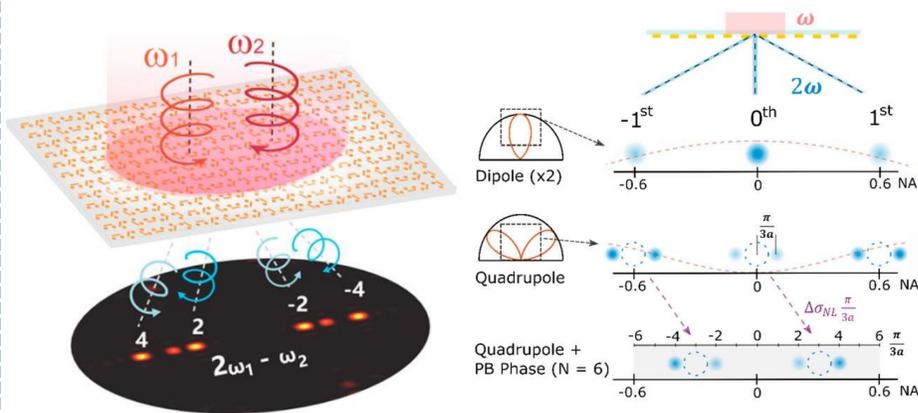
### 偶极子近似



Nanophotonics., 2018, 7(6), 1013-1024.

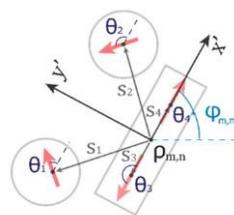
$$\mathbf{p}_{\theta,\sigma}^{n\omega} = \mathbf{p}_{\theta,L,\sigma}^{n\omega} e^{-i\sigma\theta} \propto e^{(n-1)i\sigma\theta}$$
$$\mathbf{p}_{\theta,-\sigma}^{n\omega} = \mathbf{p}_{\theta,L,-\sigma}^{n\omega} e^{i\sigma\theta} \propto e^{(n+1)i\sigma\theta}$$

### 多极子



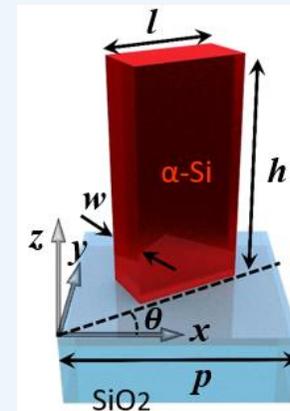
ACS Photonics., 2019, 6, 2335-2341.

所有多极子均位于x-y 平面内

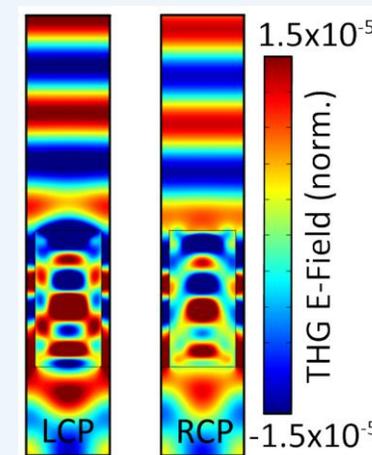
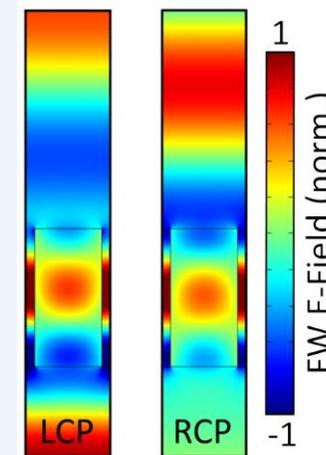


Multipoles  
Multi-dipoles

### 介质型非线性几何超表面



- 三维空间内具有复杂的多极子分布.
- z方向传播距离不为0.
- 面外多极子的辐射特性, 对总散射场贡献变大



金属型非线性几何超表面

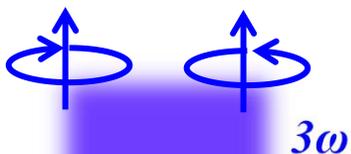


## 2. 非线性几何相位超表面

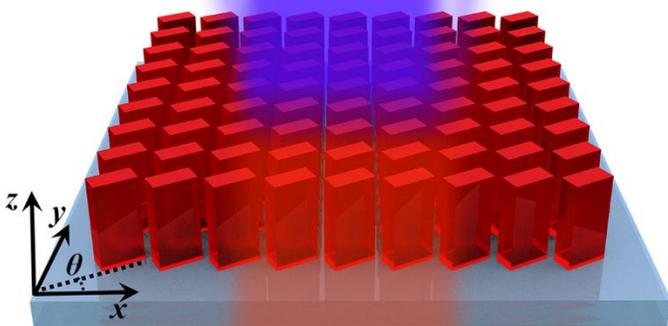


### ➤ 介质超构表面非线性光学过程的波耦合模型

TH Signals



$3\omega$



$\omega$



FW Wave

硅超构表面中的三次谐波产生

非线性极化强度:

- 1) 圆偏振坐标系(CPR);
- 2) 进行旋转操作.

$$P^{\alpha'}(3\omega) = \epsilon_0 \sum_{\beta'\gamma'\delta'} \sum_{\alpha\beta\gamma\delta} R_{\alpha}^{\alpha'} \chi_{\alpha\beta\gamma\delta}^{(3)\text{eff}} R_{\beta'}^{\beta} R_{\gamma'}^{\gamma} R_{\delta'}^{\delta} E^{\beta'} E^{\gamma'} E^{\delta'}$$

$$\chi_{\alpha'\beta'\gamma'\delta'}^{(3)} = \sum_{\alpha\beta\gamma\delta} \chi_{\alpha\beta\gamma\delta}^{(3)\text{eff}} \underbrace{R_{\alpha}^{\alpha'} R_{\beta'}^{\beta} R_{\gamma'}^{\gamma} R_{\delta'}^{\delta}}_{\text{非线性几何相位}}$$

非线性几何相位

圆偏振坐标系下的旋转矩阵元

$$R_{\alpha}^{\alpha'} = \Lambda_{\alpha'}^{i'} R_i^{\alpha} \Lambda_i^{\alpha} = \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & e^{-i\theta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{e}_{\alpha} = \sum_i \Lambda_{\alpha}^i \hat{e}_i \quad \alpha \in \{L, R, z\}$$

$$\hat{e}_i = \sum_{\alpha} \Lambda_{\alpha}^i \hat{e}_{\alpha} \quad i \in \{x, y, z\}$$

注意:

- 1) 对于单独的纳米结构, 上述表达式始终准确;
- 2) 对于周期的超原子晶格阵列, 当结构单元间的耦合不太强时, 上述表达式近似成立。



## 2. 非线性几何相位超表面



C2 硅柱阵列中的非线性极化强度



波耦合物理图像

### LCP nonlinear polarizations:

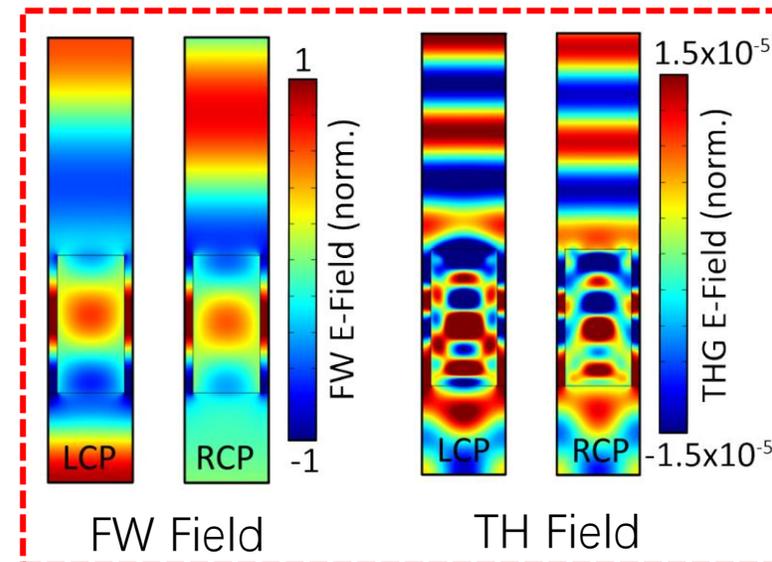
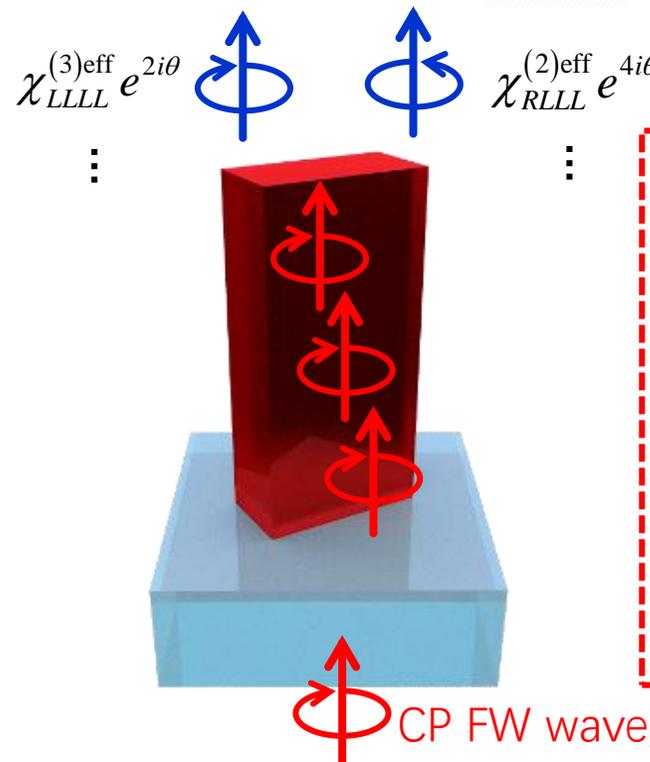
$$\begin{aligned}
P^L(\mathbf{r}, \theta, 3\omega) = & \varepsilon_0 \chi_{LLLL}^{(3)\text{eff}} e^{2i\theta} (E^L)^3 + \varepsilon_0 \chi_{LRRR}^{(3)\text{eff}} e^{-4i\theta} (E^R)^3 \\
& + \varepsilon_0 \chi_{LLzz}^{(3)\text{eff}} E^L (E^z)^2 + 3\varepsilon_0 \chi_{LLLR}^{(3)\text{eff}} E^R (E^L)^2 \\
& + \varepsilon_0 e^{-2i\theta} \left[ 3\chi_{LLRR}^{(3)\text{eff}} E^L (E^R)^2 + \chi_{LRzz}^{(3)\text{eff}} E^R (E^z)^2 \right]
\end{aligned}$$

### RCP nonlinear polarizations:

$$\begin{aligned}
P^R(\mathbf{r}, \theta, 3\omega) = & \varepsilon_0 \chi_{RRRR}^{(3)\text{eff}} e^{-2i\theta} (E^R)^3 + \varepsilon_0 \chi_{RLLL}^{(3)\text{eff}} e^{4i\theta} (E^L)^3 \\
& + \varepsilon_0 \chi_{RRzz}^{(3)\text{eff}} E^R (E^z)^2 + 3\varepsilon_0 \chi_{RRRL}^{(3)\text{eff}} E^L (E^R)^2 \\
& + \varepsilon_0 e^{2i\theta} \left[ 3\chi_{RRLL}^{(3)\text{eff}} E^R (E^L)^2 + \chi_{RLzz}^{(3)\text{eff}} E^L (E^z)^2 \right]
\end{aligned}$$

- No phase (DC term)

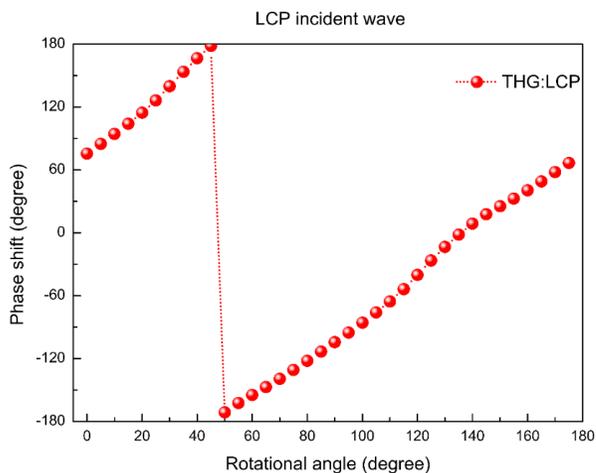
- Geometric phase (GP):  $\exp(2i\sigma\theta)$ ,  $\exp(4i\sigma\theta)$
- Conjugate GP:  $\exp(-2i\sigma\theta)$ ,  $\exp(-4i\sigma\theta)$



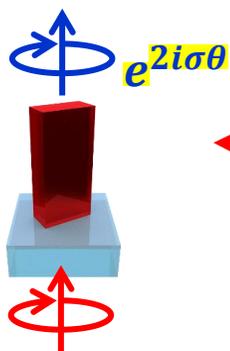
- ✓ 不同的非线性极化矢量对应着不同的波耦合过程，并导致其携带有不同的几何相位调制，最终令辐射到自由空间的谐波信号也携带有对应的得先行几何相位。



# 2. 非线性几何相位超表面



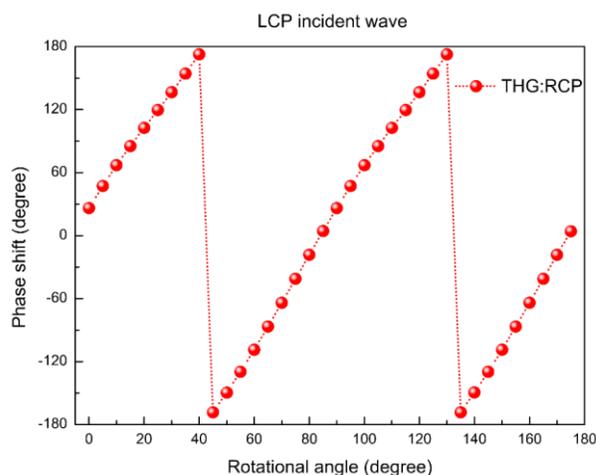
THG: LCP



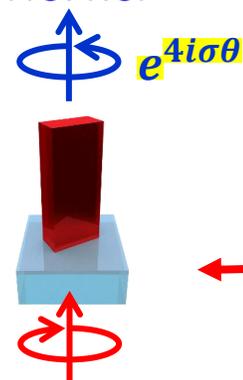
LCP FW incident, LCP TH output  
Co-polarized conversion

FW: LCP,  $\sigma = 1$

$$P_L^{(3)\text{eff}} = \epsilon_0 \left[ \chi_{LLLL}^{(3)\text{eff}} e^{2i\theta} (\bar{E}^L)^3 + \chi_{LRRR}^{(3)\text{eff}} e^{-4i\theta} (\bar{E}^R)^3 \right] + 3\epsilon_0 \left[ \chi_{LLLR}^{(3)\text{eff}} \bar{E}^R (\bar{E}^L)^2 + \chi_{LLRR}^{(3)\text{eff}} e^{-2i\theta} \bar{E}^L (\bar{E}^R)^2 \right]$$



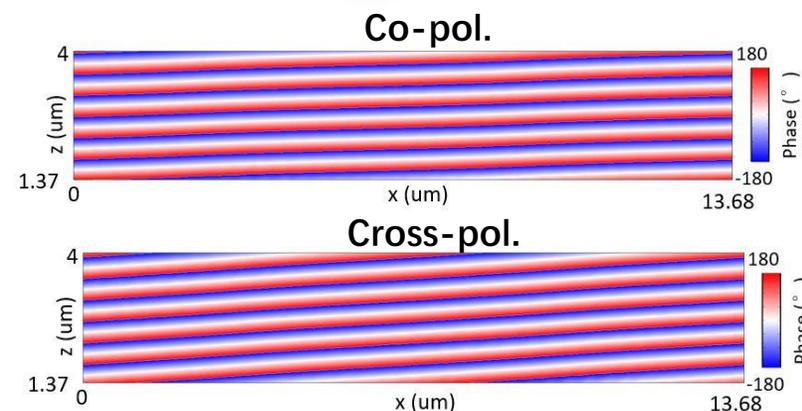
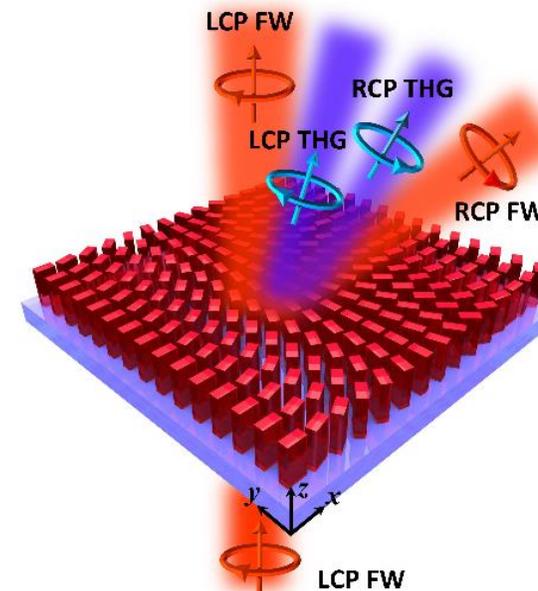
THG: RCP



LCP FW incident, RCP TH output  
Cross-polarized conversion

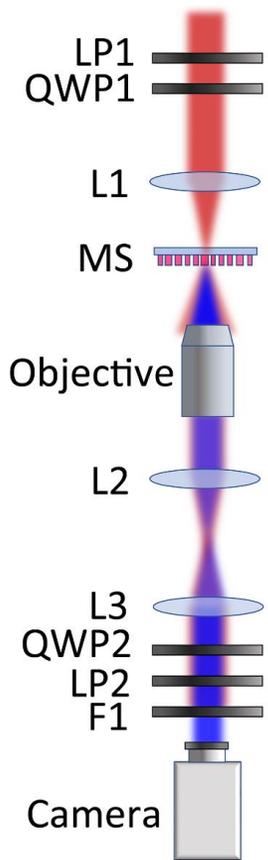
FW: LCP,  $\sigma = 1$

$$P_R^{(3)\text{eff}} = \epsilon_0 \left[ \chi_{RRRR}^{(3)\text{eff}} e^{-2i\theta} (\bar{E}^R)^3 + \chi_{RLLL}^{(3)\text{eff}} e^{4i\theta} (\bar{E}^L)^3 \right] + 3\epsilon_0 \left[ \chi_{RRRL}^{(3)\text{eff}} \bar{E}^L (\bar{E}^R)^2 + \chi_{RRLL}^{(3)\text{eff}} e^{2i\theta} \bar{E}^R (\bar{E}^L)^2 \right]$$





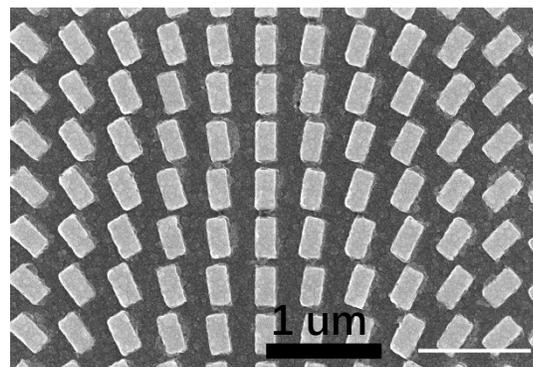
## 2. 非线性几何相位超表面



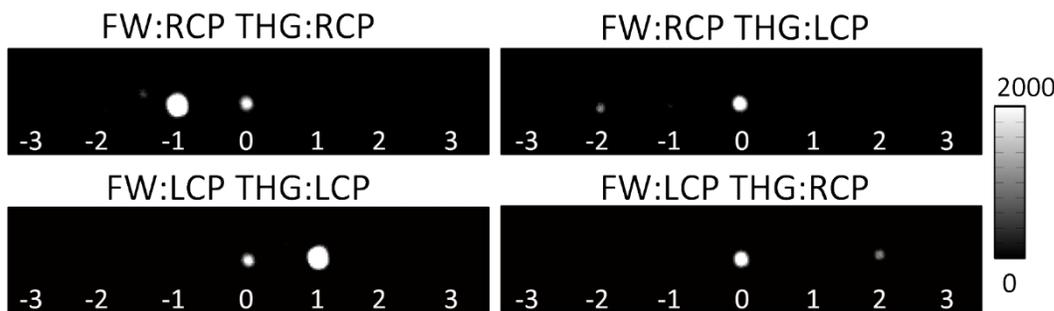
**C2 silicon  
Meta-atom**



SEM image of NL GPM



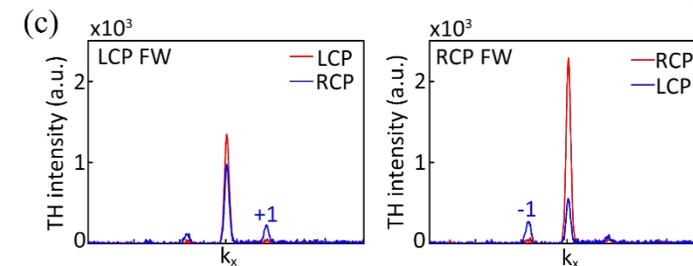
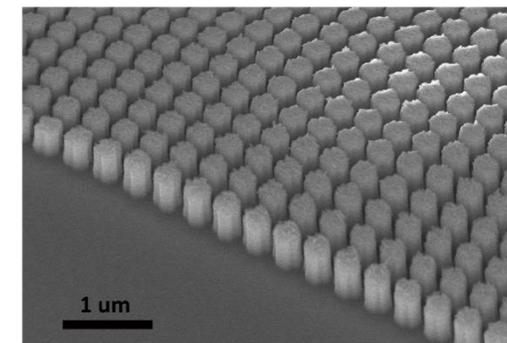
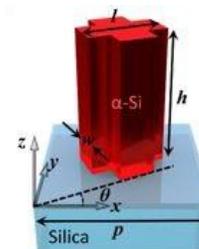
Sample: 20 units/period, FW @ 1300 nm



**Co-pol. TH**  
Theory:  $\pm 3.27^\circ$   
Exp.:  $\pm 3.06^\circ$

**Cross-pol. TH**  
Theory:  $\pm 6.55^\circ$   
Exp.:  $\pm 6.10^\circ$

**C4 silicon  
Meta-atom**



**Neumann  
principle**

$$P_L^{(3)\text{eff}} = \epsilon_0 \chi_{LRRR}^{(3)\text{eff}} e^{-4i\sigma\theta} (\bar{E}^R)^3 + 3\epsilon_0 \chi_{LLLL}^{(3)\text{eff}} \bar{E}^R (\bar{E}^L)^2$$

$$P_R^{(3)\text{eff}} = \epsilon_0 \chi_{RLLL}^{(3)\text{eff}} e^{4i\sigma\theta} (\bar{E}^L)^3 + 3\epsilon_0 \chi_{RRRL}^{(3)\text{eff}} \bar{E}^L (\bar{E}^R)^2$$



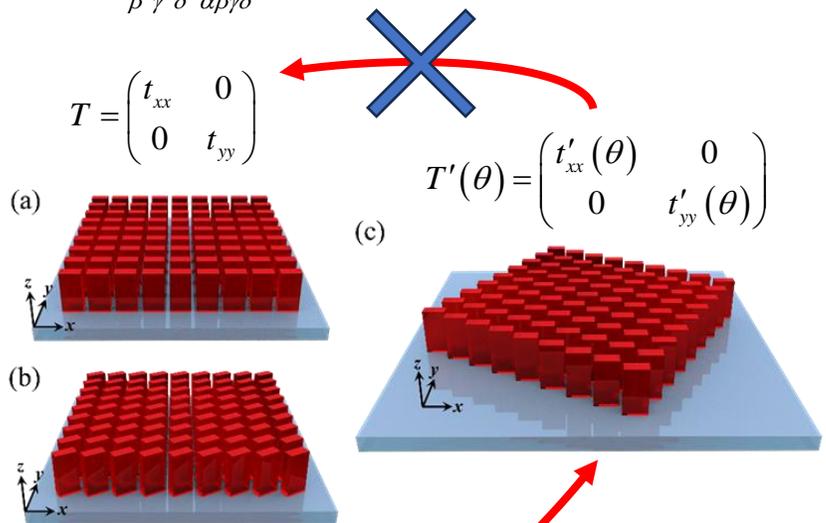
## 2. 非线性几何相位超表面



一个一般性的问题:

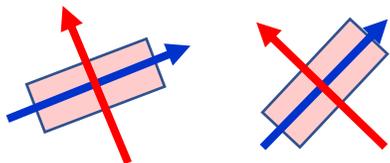
结构单元间耦合将会给非线性几何相位调制带来什么影响?

$$P^{\alpha'}(3\omega) = \epsilon_0 \sum_{\beta'\gamma'\delta'} \sum_{\alpha\beta\gamma\delta} R_{\alpha}^{\alpha'} \chi_{\alpha\beta\gamma\delta}^{(3)\text{eff}} R_{\beta}^{\beta'} R_{\gamma}^{\gamma'} R_{\delta}^{\delta'} E^{\beta'} E^{\gamma'} E^{\delta'}$$



$$T(\theta) = \begin{pmatrix} t_{xx}(\theta) & t_{xy}(\theta) \\ t_{yx}(\theta) & t_{yy}(\theta) \end{pmatrix}$$

Principal axis transformation



$$n_{fast}^{eff}(\theta) \quad n_{slow}^{eff}(\theta)$$

人造晶格体系中非线性光学过程的选择定则



	C2	C3	C4	C5
2 <sup>nd</sup> harmonic generation				
3 <sup>rd</sup> harmonic generation	2θ (σ) 4θ (-σ) ✓	3θ (-σ) ?	4θ (-σ) ✓	?
4 <sup>th</sup> harmonic generation		3θ (σ)		5θ (-σ)
5 <sup>th</sup> harmonic generation	4θ (σ) 6θ (-σ)		4θ (σ)	
6 <sup>th</sup> harmonic generation				5θ (σ)

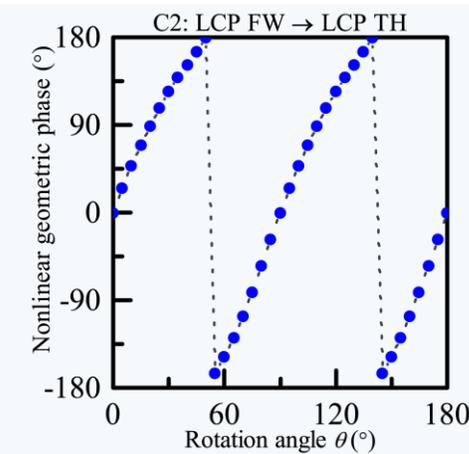
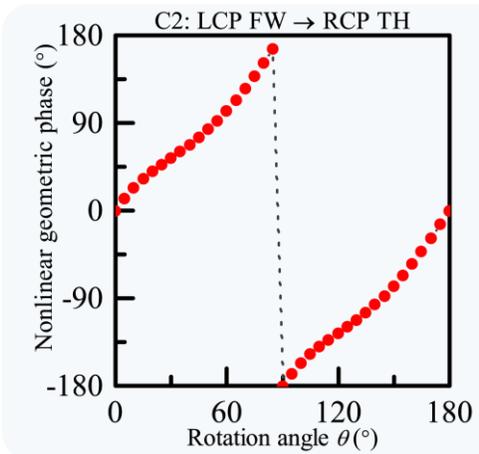
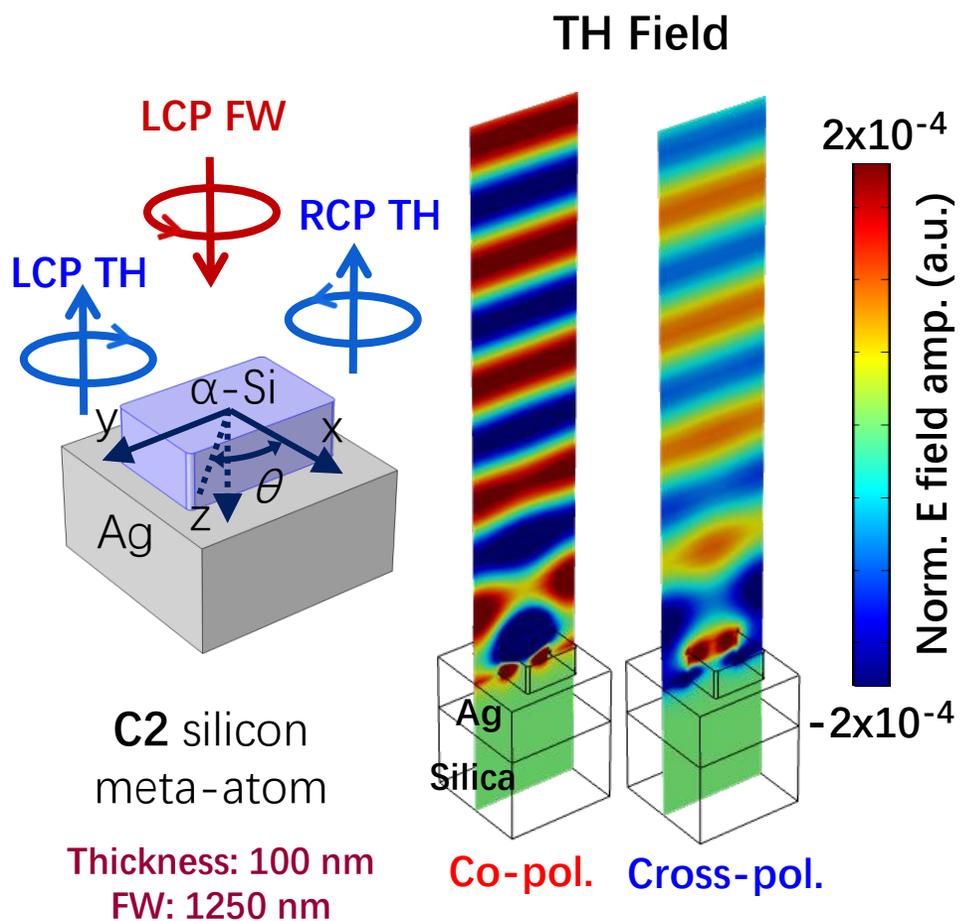
针对透射式的C3和C5结构，我们观察到了明显的非线性几何相位调制，但是三次谐波的转换效率非常低！



## 2. 非线性几何相位超表面

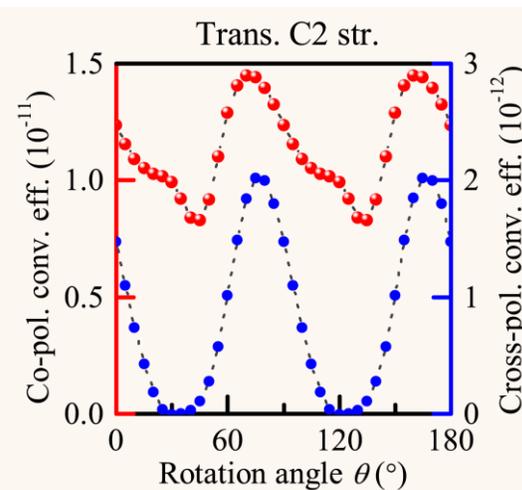
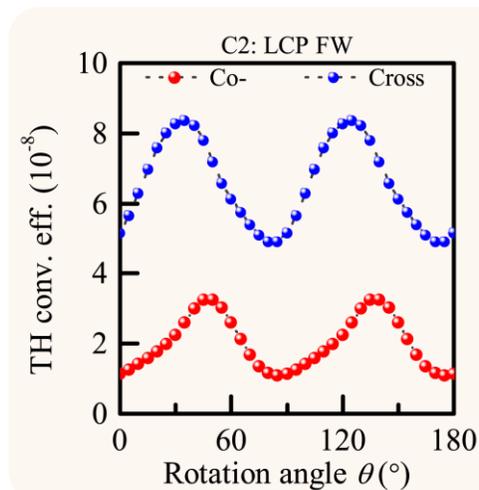


### ➤ 正方晶格中C2硅纳米柱



### Nonlinear geometric phase

- ✓ Co-polarized:  
 $\exp(2i\sigma\theta)$
- ✓ Cross-polarized:  
 $\exp(4i\sigma\theta)$



### Averaged TH conversion efficiency

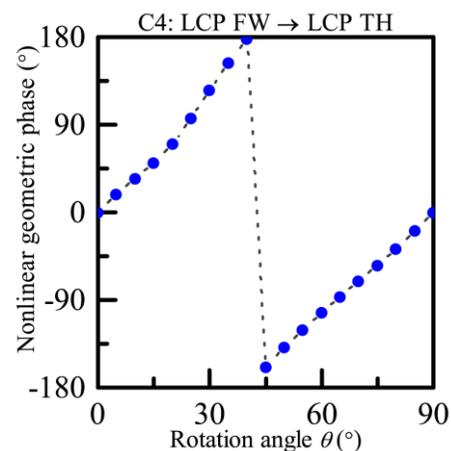
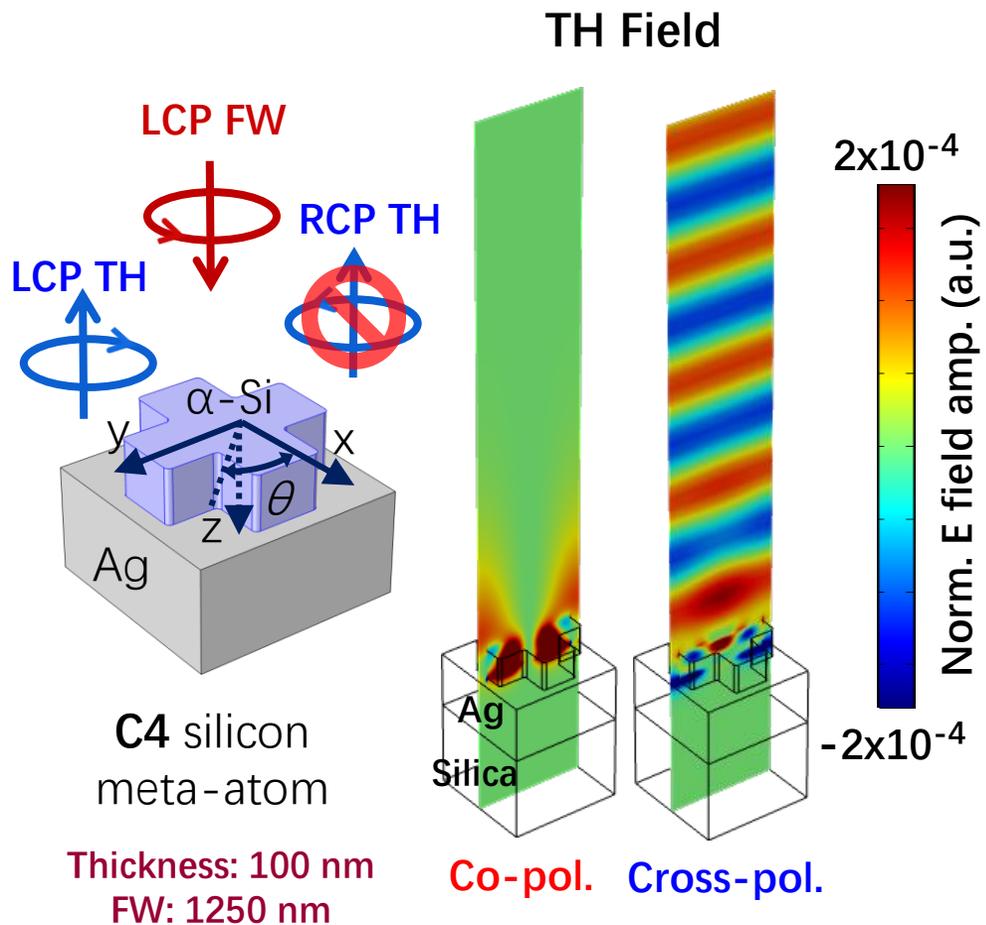
- ✓ Refl.  
 $6.5 \times 10^{-8}$ ,  $2 \times 10^{-8}$
  - ✓ Trans.  
 $1.1 \times 10^{-11}$ ,  $8.6 \times 10^{-13}$
- Improved by:  
 $10^3 \sim 10^4$



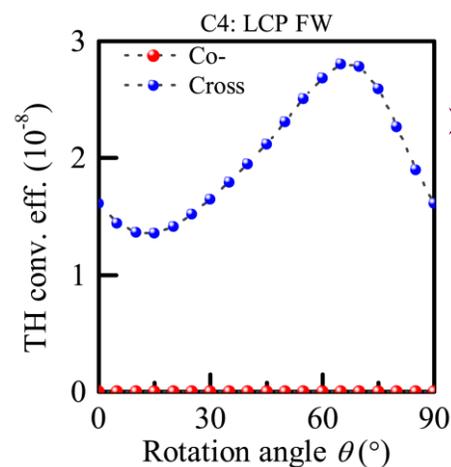
## 2. 非线性几何相位超表面



### ➤ 正方晶格中C4硅纳米柱



➤ Nonlinear geometric phase:  
Cross-pol.  
 $\exp(4i\sigma\theta)$



➤ Averaged TH conversion efficiency:  
 $4.3 \times 10^{-15}$ ,  
 $2 \times 10^{-8}$

➤ The reflective the silicon meta-atom follows the selection rule governed by the structure symmetry

GP of TH signals

Co-pol. Cross-pol.

C1:  $2\sigma\theta$   $4\sigma\theta$

C2:  $2\sigma\theta$   $4\sigma\theta$

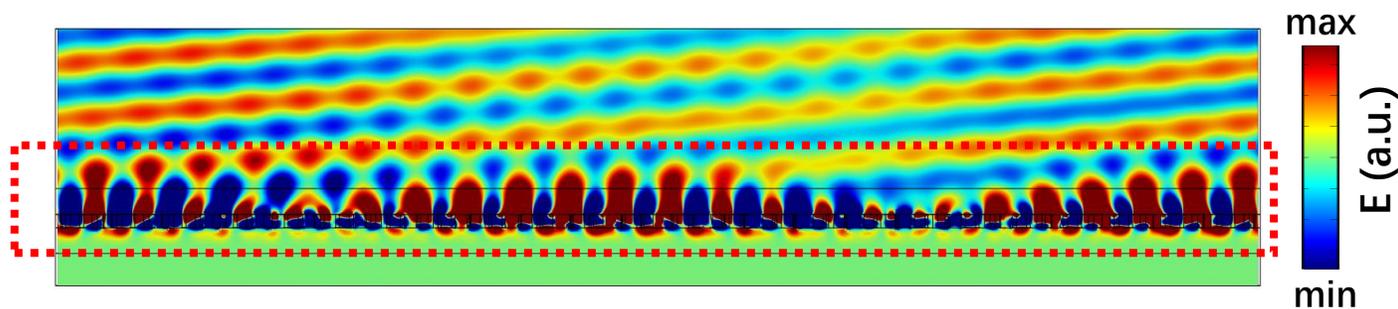
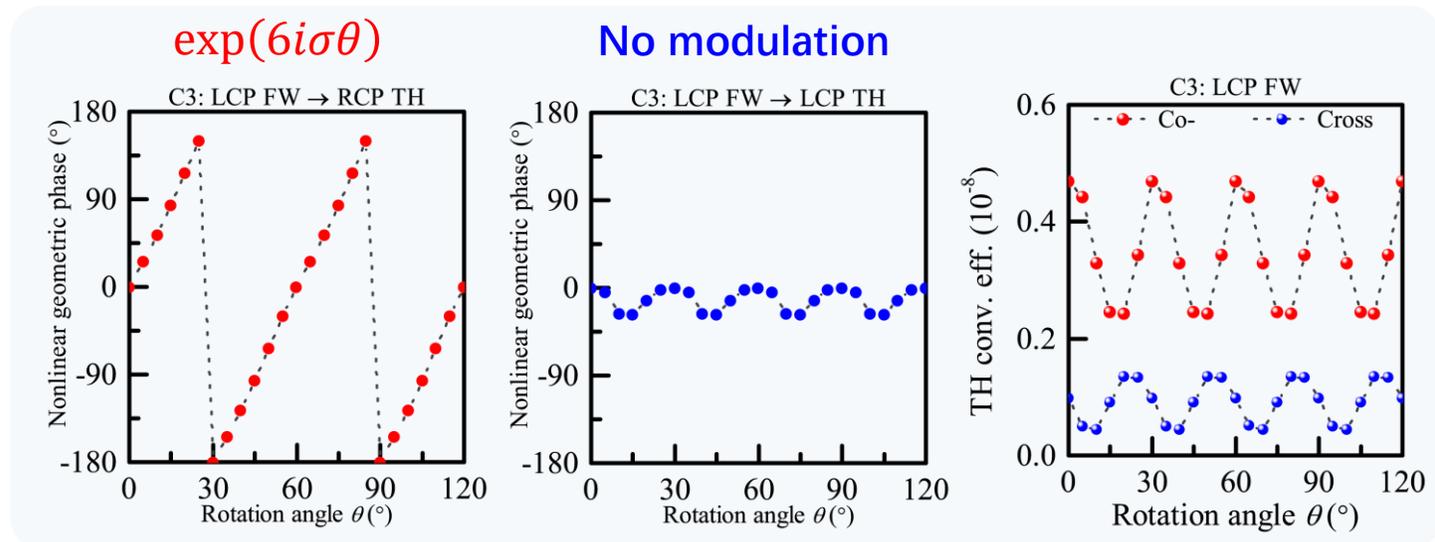
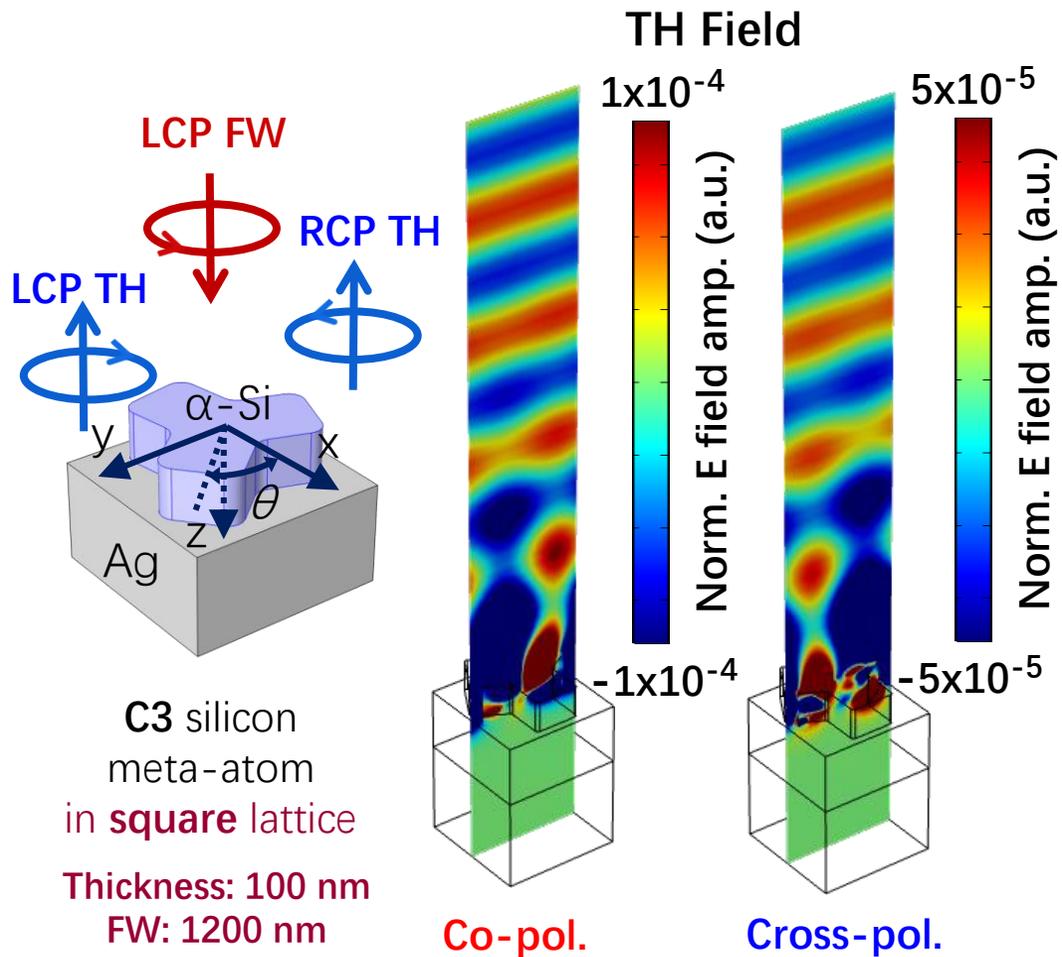
C4:  $\times$   $4\sigma\theta$



## 2. 非线性几何相位超表面



➤ 正方晶格中C3硅纳米柱



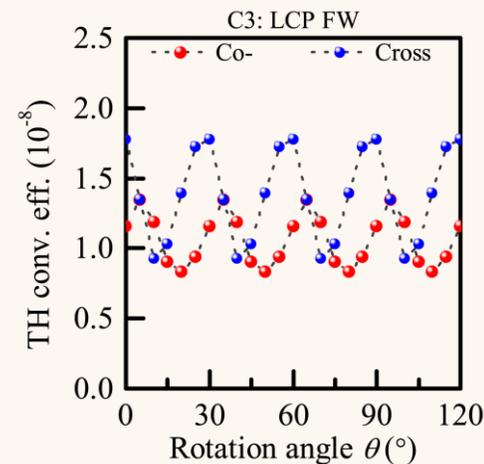
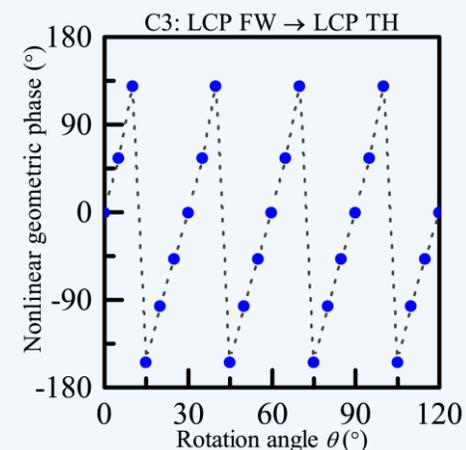
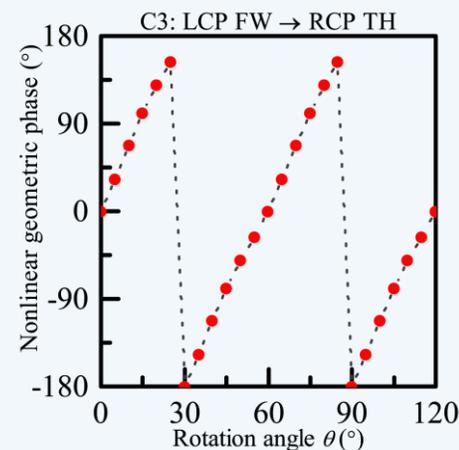
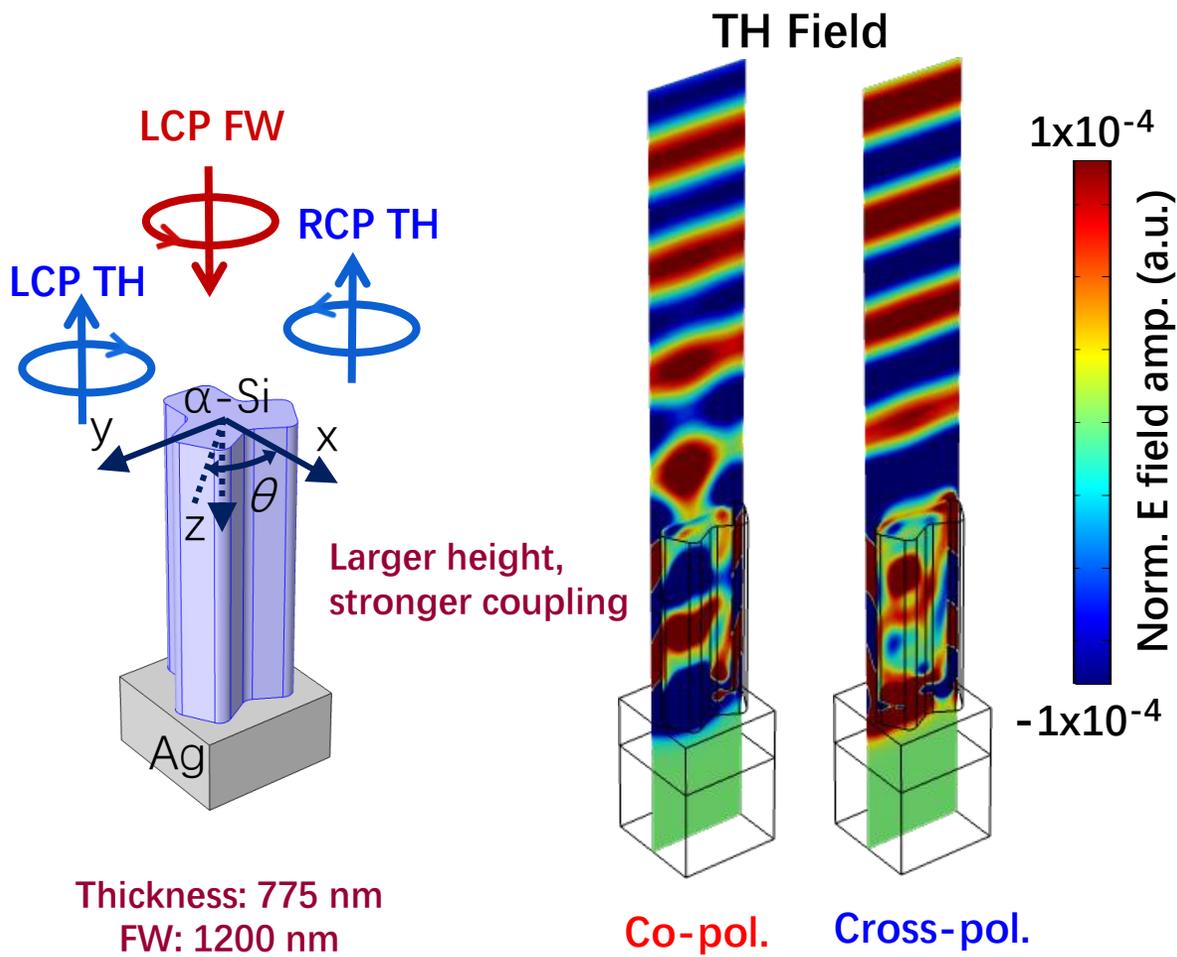
基于C3结构的梯度超表面：三次谐波信号的反常反射



## 2. 非线性几何相位超表面



➤ 正方晶格中C3硅纳米柱：改变结构间的耦合强度



Nonlinear geometric phase

✓ Co-polarized:

$$\exp(6i\sigma\theta)$$

✓ Cross-polarized:

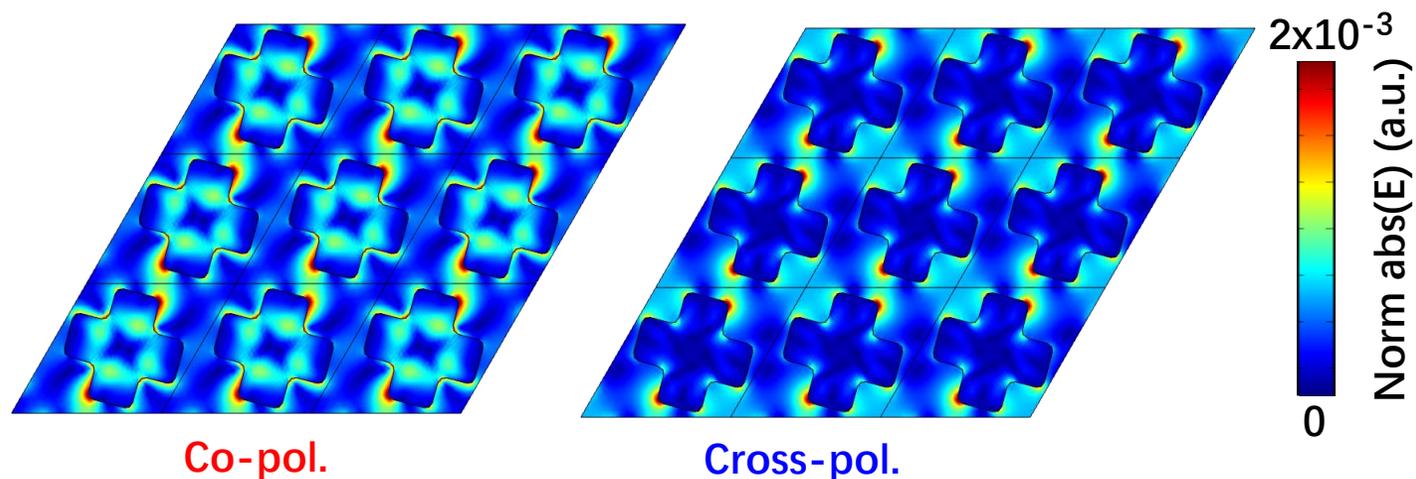
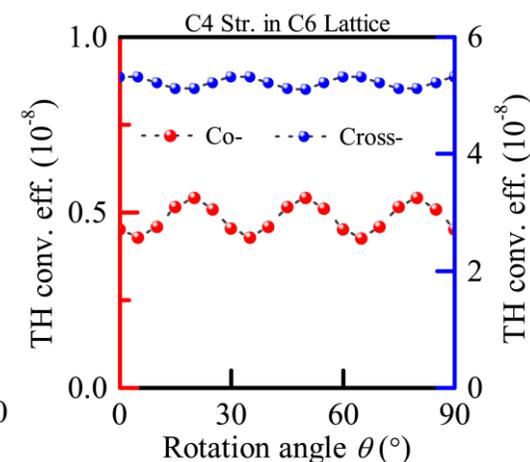
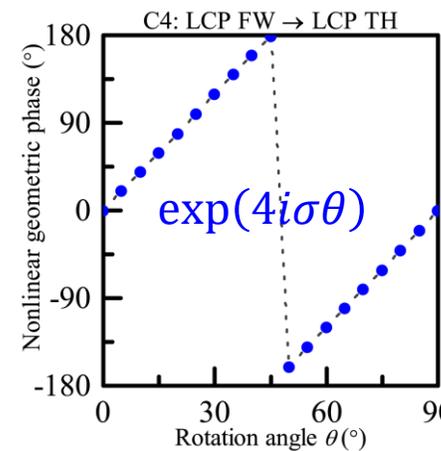
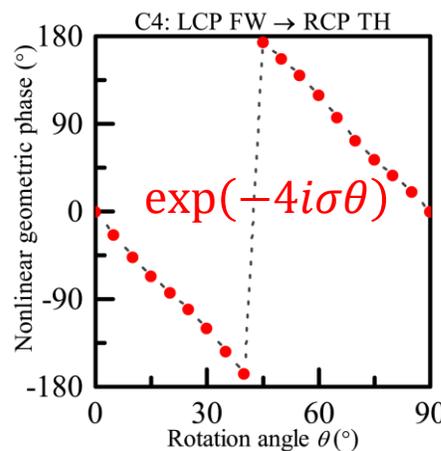
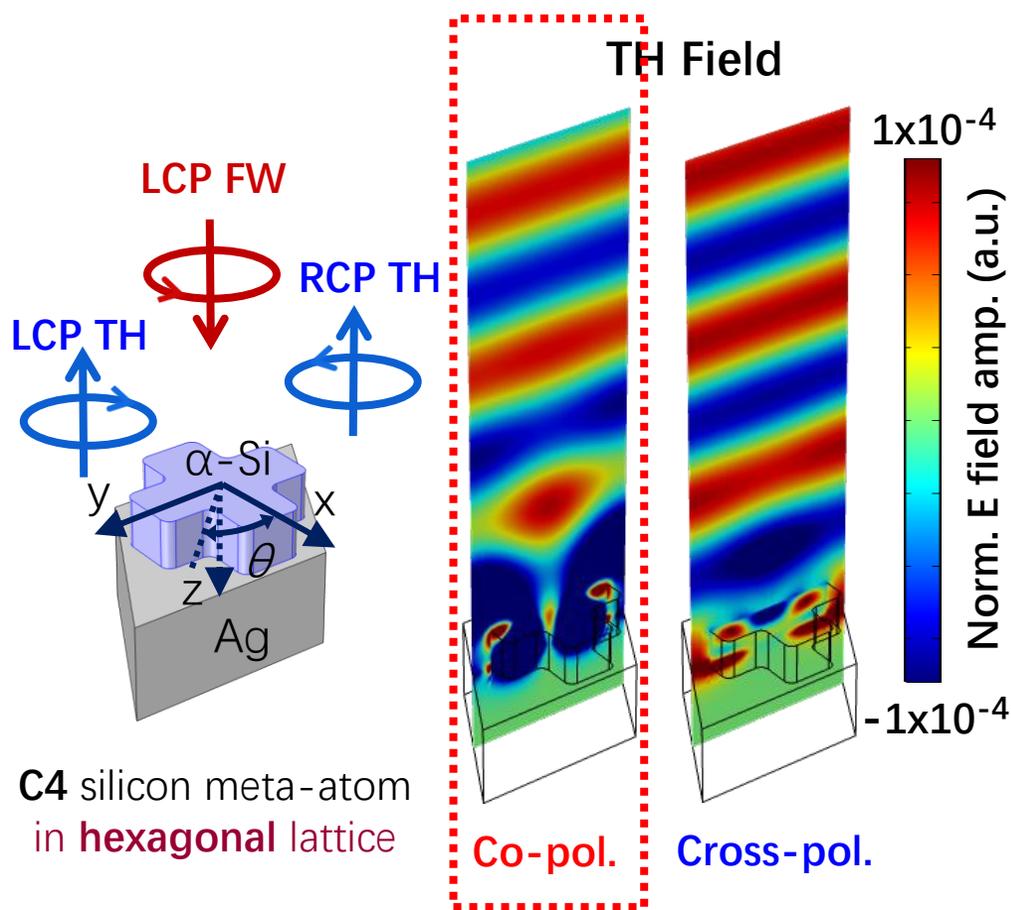
$$\exp(12i\sigma\theta)$$



## 2. 非线性几何相位超表面



➤ 六角晶格中C6硅纳米柱





## 2. 非线性几何相位超表面



	C2	C3	C4	C5
2 <sup>nd</sup> harmonic generation				
3 <sup>rd</sup> harmonic generation	2θ (σ) 4θ (-σ)	3θ (-σ)	4θ (-σ)	
4 <sup>th</sup> harmonic generation		3θ (σ)		5θ (-σ)
5 <sup>th</sup> harmonic generation	4θ (σ) 6θ (-σ)	6θ (-σ)	4θ (σ)	
6 <sup>th</sup> harmonic generation				5θ (σ)

表1. 正方及六角晶格中C1-C5 硅纳米柱三次谐波的几何相位

Lattice Symmetry	Pol. Conv.	C1	C2	C3	C4	C5
Square	Co-pol.	$2\sigma\theta$	$2\sigma\theta$	$6\sigma\theta$		$\pm 10\sigma\theta$
	Cross-pol.	$4\sigma\theta$	$4\sigma\theta$	$12\sigma\theta$	$4\sigma\theta$	-
Hexagonal	Co-pol.	$2\sigma\theta$	$2\sigma\theta$		$-4\sigma\theta$	$-10\sigma\theta$
	Cross-pol.	$4\sigma\theta$	$4\sigma\theta$		$4\sigma\theta$	$10\sigma\theta$

人造晶格体系中非线性光学过程的选择定则



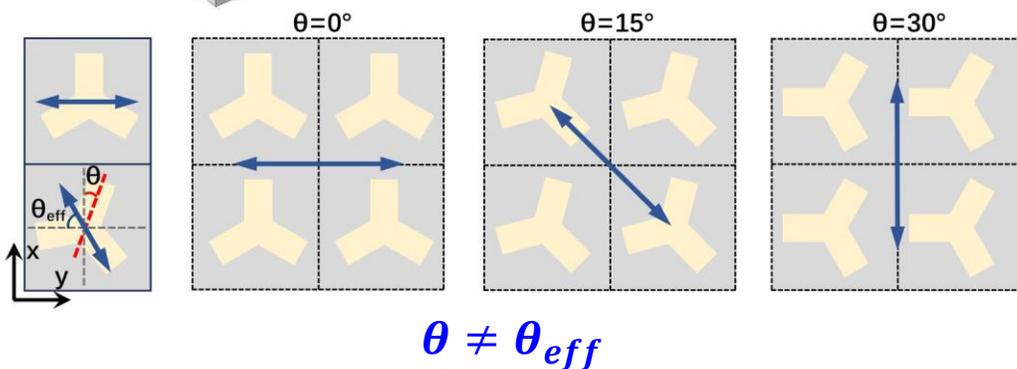
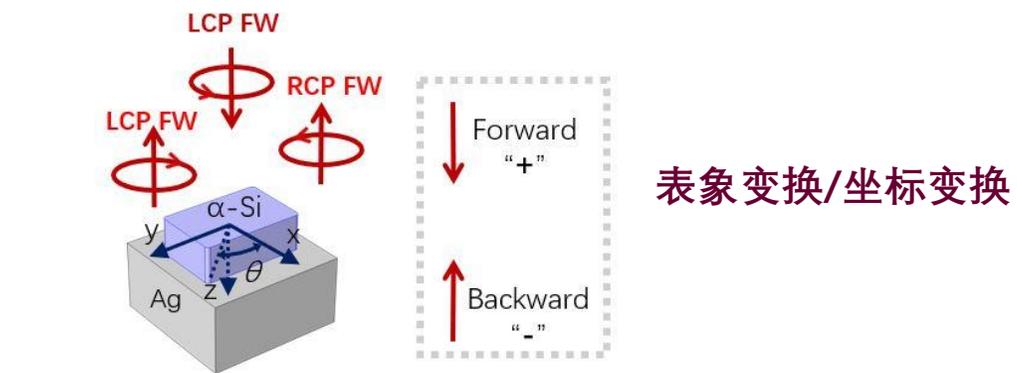
## 2. 非线性几何相位超表面



### ➤ 线性及非线性几何相位产生的根本原因

$$P^{\alpha'}(3\omega) = \varepsilon_0 \sum_{\alpha, \beta} -R_{\alpha}^{\alpha'}(\theta_{eff}) \chi_{\alpha\beta\gamma\delta}^{(3)} \left[ +R_{\beta}^{\beta'}(\theta_{eff}) \right]^{-1} \left[ +R_{\gamma}^{\gamma'}(\theta_{eff}) \right]^{-1} \left[ +R_{\delta}^{\delta'}(\theta_{eff}) \right]^{-1} \boxed{E^{\beta}(\omega) E^{\gamma}(\omega) E^{\delta}(\omega)} \rightarrow \text{Modes of meta-atom @ FW}$$

$$\chi_{\alpha'\beta'\gamma'\delta'}^{(3)}(\theta_{eff}) = \sum_{\alpha, \beta, \gamma, \delta} -R_{\alpha}^{\alpha'} \chi_{\alpha\beta\gamma\delta}^{(3)} \left[ +R_{\beta}^{\beta'} \right]^{-1} \left[ +R_{\gamma}^{\gamma'} \right]^{-1} \left[ +R_{\delta}^{\delta'} \right]^{-1}$$



- 基频光波长远大于结构周期(波长 ~ 3倍周期): **非局域效应的贡献变大**, 单元间耦合效应明显, 即此时结构内部的基频场分布接近于**等效介质薄膜**中的波导模式.
- 结构旋转导致等效介质薄膜的光轴发生相应的旋转.
- 结构单元间的耦合导致:  $\theta \neq \theta_{eff}$

确定  $\theta$  与  $\theta_{eff}$  之间的关系: **广义线性几何相位**

PHYSICAL REVIEW LETTERS 126, 183902 (2021)

Generalized Pancharatnam-Berry Phase in Rotationally Symmetric Meta-Atoms

Xin Xie<sup>1,2,3</sup>, Mingbo Pu<sup>1,2</sup>, Jinjin Jin<sup>1</sup>, Mingfeng Xu<sup>1,4</sup>, Yinghui Guo<sup>1,2</sup>, Xiong Li<sup>1,2</sup>, Ping Gao<sup>1</sup>, Xiaoliang Ma<sup>1,2</sup> and Xiangang Luo<sup>1,2,\*</sup>





## 2. 非线性几何相位超表面



✓ 确定  $\theta$  和  $\theta_{eff}$  之间的关系 线性几何相位的波耦合模型推导:  $\chi_{\alpha'\beta'}^{(1)} = \sum_{\alpha,\beta} -R_{\alpha}^{\alpha'}(\theta_{eff}) \chi_{\alpha\beta}^{(1)} [{}^+R_{\beta}^{\beta'}(\theta_{eff})]^{-1}$

Cross-polarization conversion process:

$$\chi_{L'L'}^{(1)} = -R_L^{L'}(\theta_{eff}) \chi_{LL}^{(1)} [{}^+R_L^{L'}(\theta_{eff})]^{-1} = \chi_{LL}^{(1)} \exp(j2\theta_{eff})$$

$$\chi_{R'R'}^{(1)} = -R_R^{R'}(\theta_{eff}) \chi_{RR}^{(1)} [{}^+R_R^{R'}(\theta_{eff})]^{-1} = \chi_{RR}^{(1)} \exp(-j2\theta_{eff})$$

Co-polarization conversion process:

$$\chi_{R'L'}^{(1)} = -R_R^{R'}(\theta_{eff}) \chi_{RL}^{(1)} [{}^+R_L^{L'}(\theta_{eff})]^{-1} = \chi_{RL}^{(1)}$$

$$\chi_{L'R'}^{(1)} = -R_L^{L'}(\theta_{eff}) \chi_{LR}^{(1)} [{}^+R_R^{R'}(\theta_{eff})]^{-1} = \chi_{LR}^{(1)}$$

$$\chi_{L'L'}^{(1)} = -R_L^{L'}(\theta_{eff}) \chi_{LL}^{(1)} [{}^+R_L^{L'}(\theta_{eff})]^{-1} = \chi_{LL}^{(1)} \exp(j2\theta_{eff}) = \chi_{LL}^{(1)} \exp(j\Phi)$$

等效光轴的旋转角度.

线性几何相位

可以通过总的线性几何相位调制反推出等效光轴的旋转角度  $\theta_{eff} = \frac{1}{2}\Phi$



## 2. 非线性几何相位超表面



$$\theta_{eff} = \frac{1}{2}\Phi, \Phi = 2l\theta \quad \longrightarrow \quad \theta_{eff} = l\theta \quad \longleftarrow$$

### 广义线性几何相位

Lattice type	Structure symmetry type				
	C1	C2	C3	C4	C5
Square	$2\sigma\theta$	$2\sigma\theta$	$6\sigma\theta$		$10\sigma\theta$
Hexagonal	$2\sigma\theta$	$2\sigma\theta$		$-4\sigma\theta$	$-10\sigma\theta$

Cross-polarization conversion process:

$$\chi_{L'L'L'L'}^{(3)} = -R_L^{L'} \chi_{LLLL}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} = \chi_{LLLL}^{(3)} \exp(j4l\theta)$$

$$\chi_{L'L'L'R'}^{(3)} = -R_L^{L'} \chi_{LLLR}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{LLLR}^{(3)} \exp(j2l\theta)$$

$$\chi_{L'L'R'R'}^{(3)} = -R_L^{L'} \chi_{LLRR}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{LLRR}^{(3)}$$

$$\chi_{L'R'R'R'}^{(3)} = -R_L^{L'} \chi_{LRRR}^{(3)} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{LRRR}^{(3)} \exp(-j2l\theta)$$

Co-polarization conversion process:

$$\chi_{R'L'L'L'}^{(3)} = -R_R^{R'} \chi_{RLLL}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} = \chi_{RLLL}^{(3)} \exp(j2l\theta)$$

$$\chi_{R'L'L'R'}^{(3)} = -R_R^{R'} \chi_{RLLR}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_L^{L'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{RLLR}^{(3)}$$

$$\chi_{R'L'R'R'}^{(3)} = -R_R^{R'} \chi_{RLRR}^{(3)} [{}^+R_L^{L'}]^{-1} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{RLRR}^{(3)} \exp(-j2l\theta)$$

$$\chi_{R'R'R'R'}^{(3)} = -R_R^{R'} \chi_{RRRR}^{(3)} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} [{}^+R_R^{R'}]^{-1} = \chi_{RRRR}^{(3)} \exp(-j4l\theta)$$

### 广义非线性几何相位

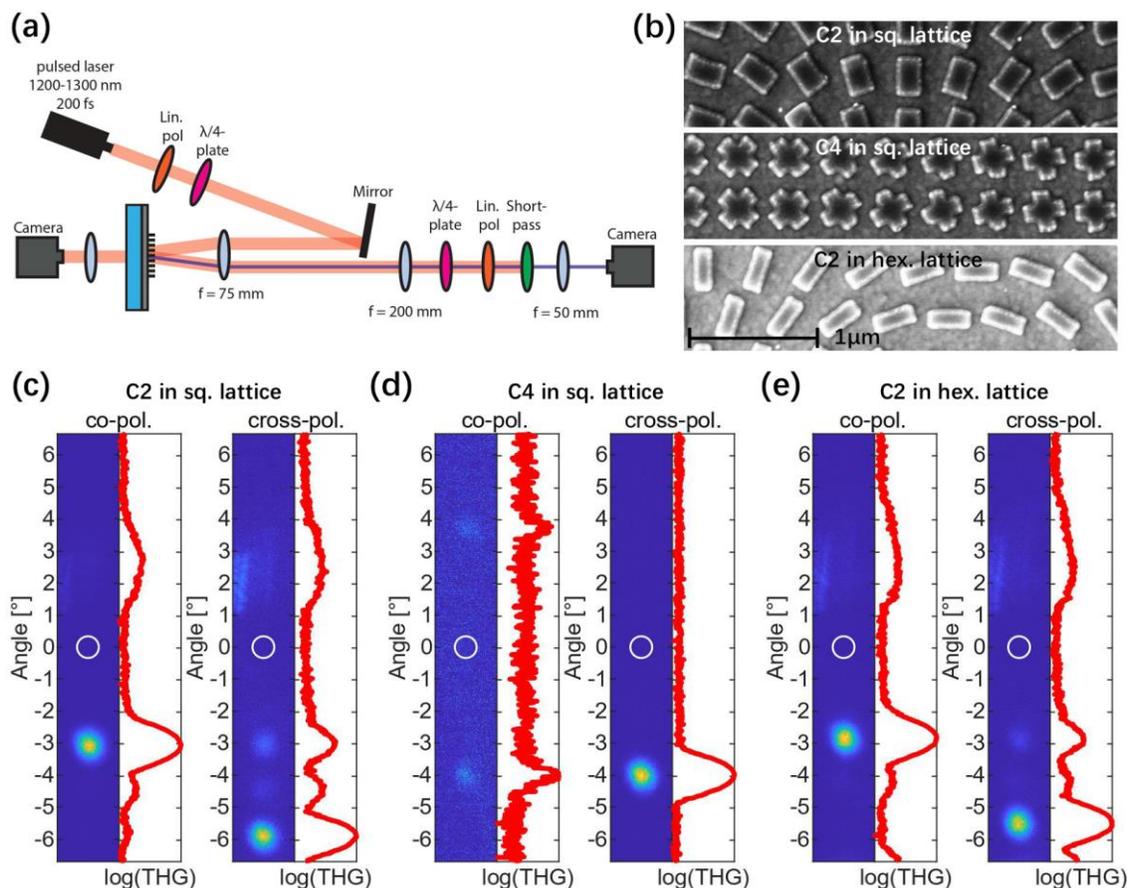
Lattice type	Pol.	Structure symmetry type				
		C1	C2	C3	C4	C5
Square	Co-pol.	$2\sigma\theta$	$2\sigma\theta$	$6\sigma\theta$		$\pm 10\sigma\theta$
	Cross-pol.	$4\sigma\theta$	$4\sigma\theta$	$12\sigma\theta$	$4\sigma\theta$	-
Hexagonal	Co-pol.	$2\sigma\theta$	$2\sigma\theta$		$-4\sigma\theta$	$-10\sigma\theta$
	Cross-pol.	$4\sigma\theta$	$4\sigma\theta$		$4\sigma\theta$	$10\sigma\theta$



## 2. 非线性几何相位超表面

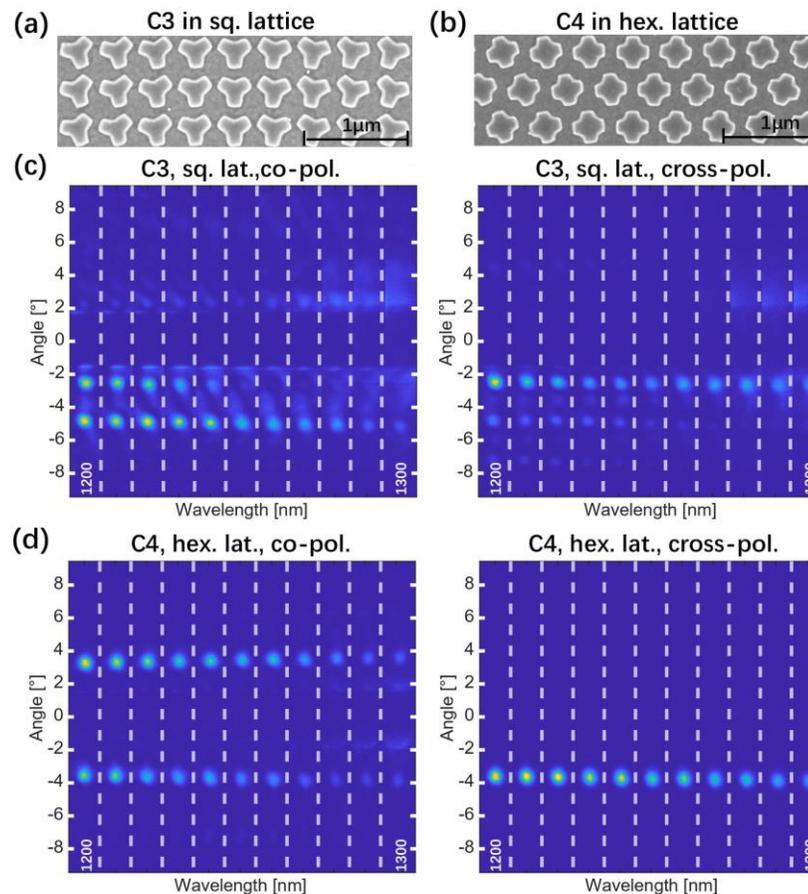


### 实验验证



由选择定则可知:

- C3结构应该无法将三次谐波信号辐射到远场.
- C4结构应无法观察到任何同偏振的三次谐波信号



实际:

- 1) C3结构可以将三次谐波信号有效地辐射到远场;
- 2) C4结构可以观察到明显的同偏振三次谐波信号

Bingyi Liu, et al. *Nonlinear dielectric geometric-phase metasurface with simultaneous structure and lattice symmetry design*, Accepted by ACS Photonics.



# 目录

1. 研究背景
2. 非线性几何相位超表面：广义非线性几何相位
- 3. 声学几何相位型超构表面**
4. 总结

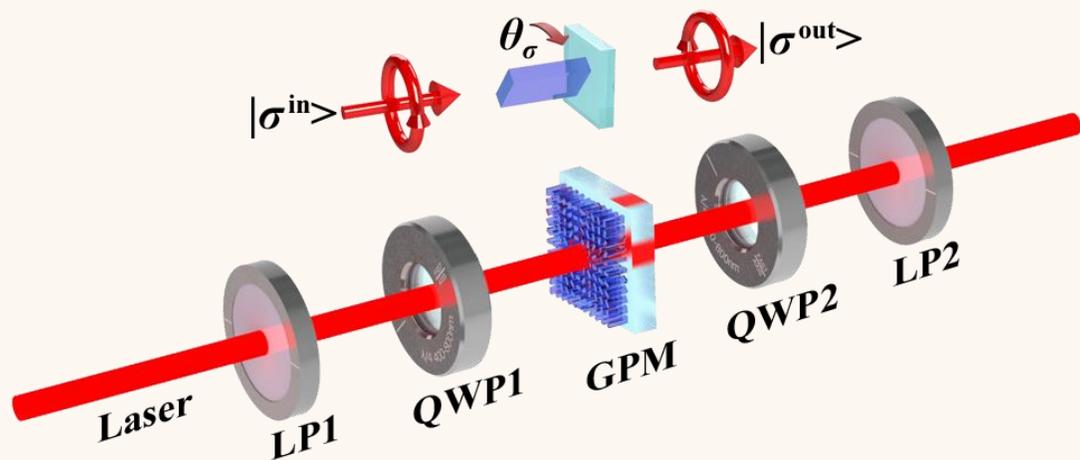


### 3. 声学几何相位型超构表面



是否可以发展出声学几何相位的概念？如何实现？

#### 电磁几何相位超构表面

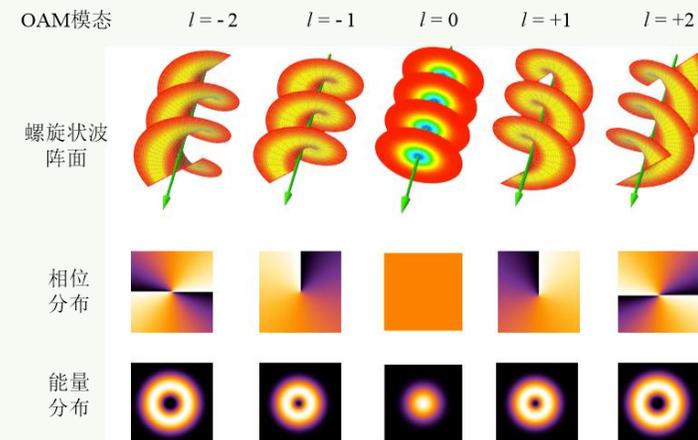


电磁几何相位:  $\exp[i(\sigma^{in} - \sigma^{out})\theta_{\sigma}] \rightarrow$  偏振转换

#### 声学几何相位超原子 (geometric-phase meta-atom)

- 自由空间声波是标量波，并没有自旋或者偏振自由度
- 可否找到一个合适的自由度，用于实现声波相位操控？

- 轨道角动量自由度！
- 不同声涡旋间的模式转换类比电磁波的偏振转换！

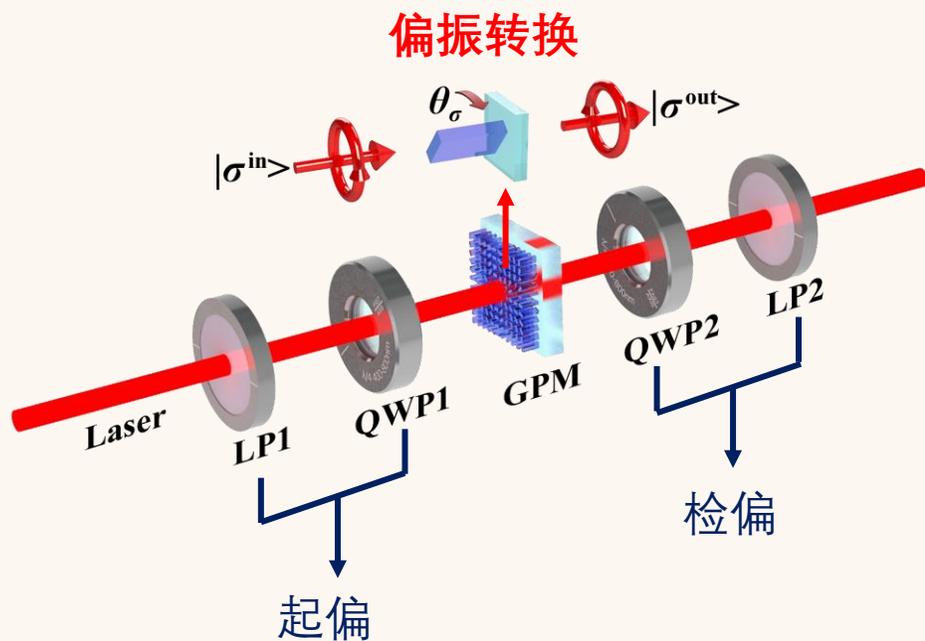




### 3. 声学几何相位型超构表面



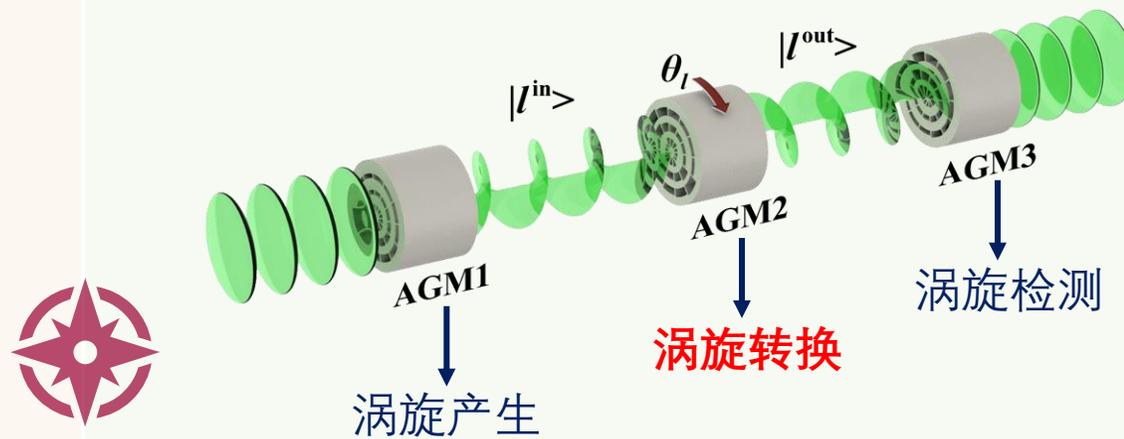
#### 电磁几何相位超构表面



Jones 矩阵:  $E_t = \hat{T}(\theta_\sigma) \begin{bmatrix} 1 \\ \sigma i \end{bmatrix} = ie^{i2\sigma\theta_\sigma} \begin{bmatrix} 1 \\ -\sigma i \end{bmatrix}$

电磁几何相位:  $\exp[i(\sigma^{in} - \sigma^{out})\theta_\sigma]$

#### 声学几何相位超原子



- 波导中声学涡旋解析表达式:  $|l(r, \phi)\rangle = J_l(k_r r) e^{il\phi} e^{ik_z z}$
- 旋转操作下声学涡旋解析表达式:  $|l(r', \phi')\rangle = \hat{R}|l(r, \phi)\rangle, r' = r, \phi' = \phi - \theta_l$   
 $|l(r', \phi')\rangle = e^{-il\theta_l}|l(r, \phi)\rangle$

声学结构透射函数:  $\hat{T}(\theta_i + \theta_l) = \hat{T}(\theta_i) \exp[i(l^{in} - l^{out})\theta_l]$

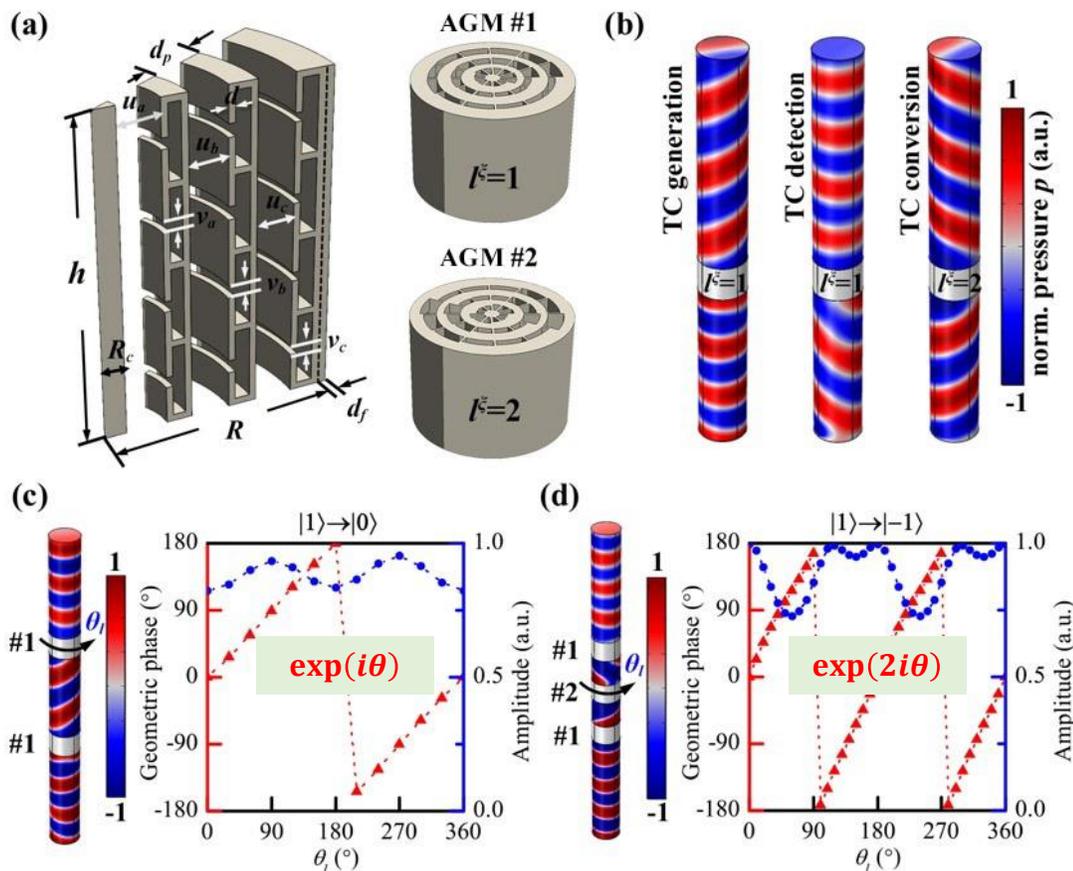
声学几何相位:  $\exp[i(l^{in} - l^{out})\theta_l]$



# 3. 声学几何相位型超构表面

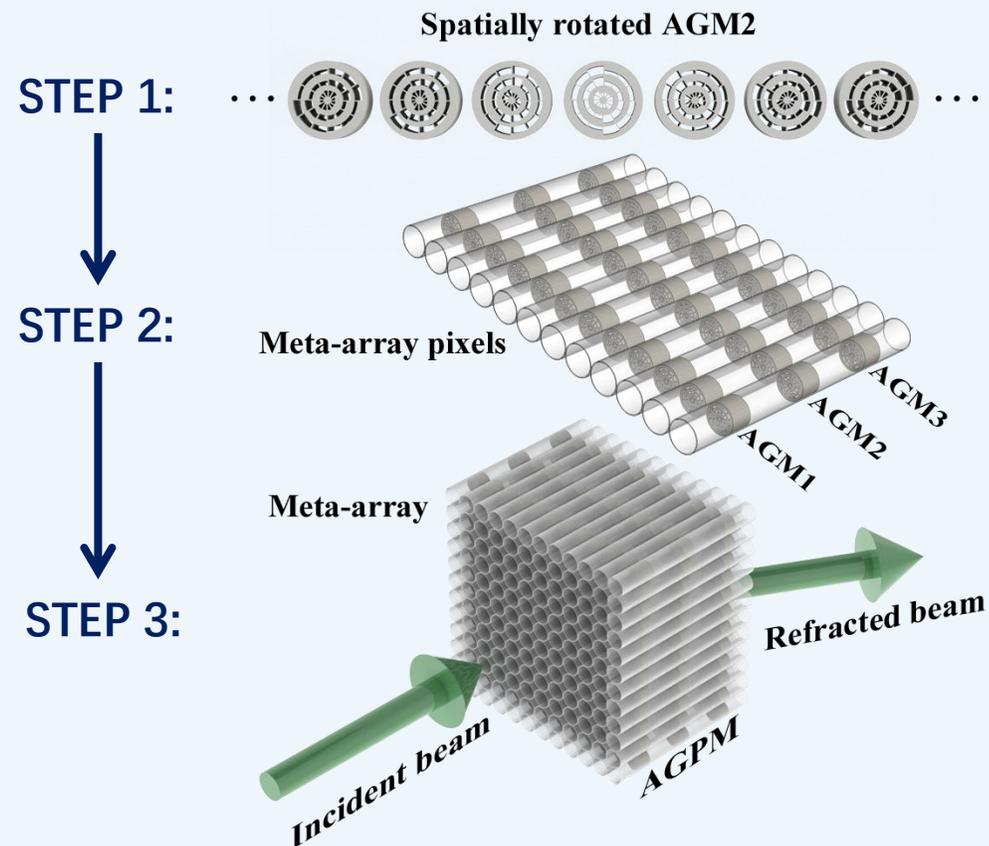


## 结构设计及仿真验证



Bingyi Liu, et al. *New Journal of Physics*, 2021, 23(11), 113026.

## ◆ 声学几何相位器件设计流程图



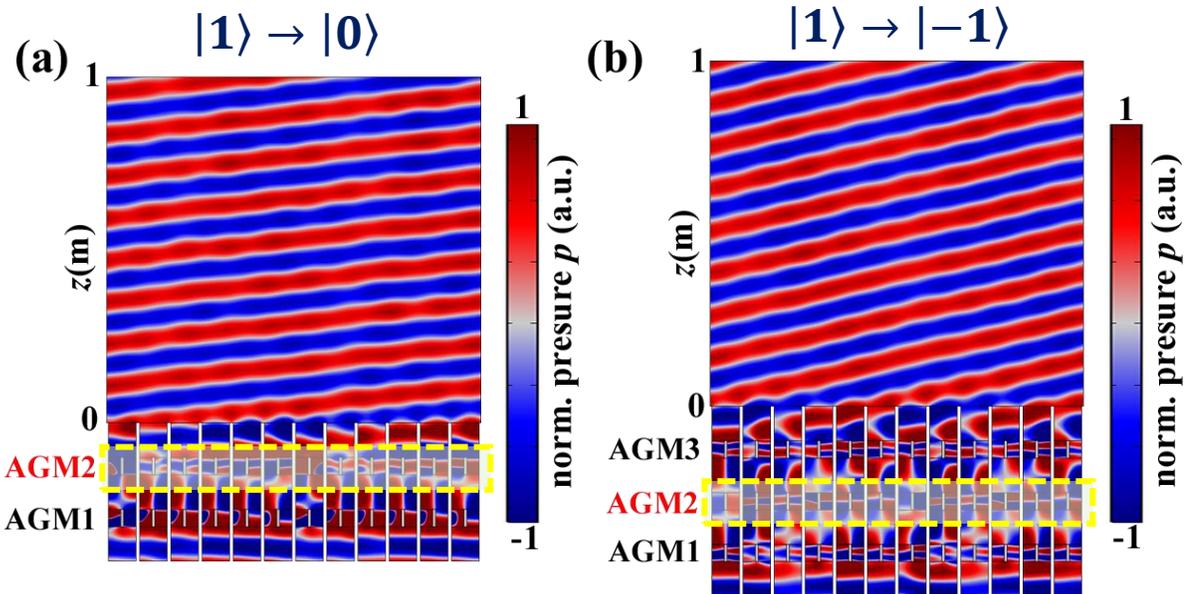
AGPM: Acoustic Geometric-Phase Meta-array



### 3. 声学几何相位型超构表面



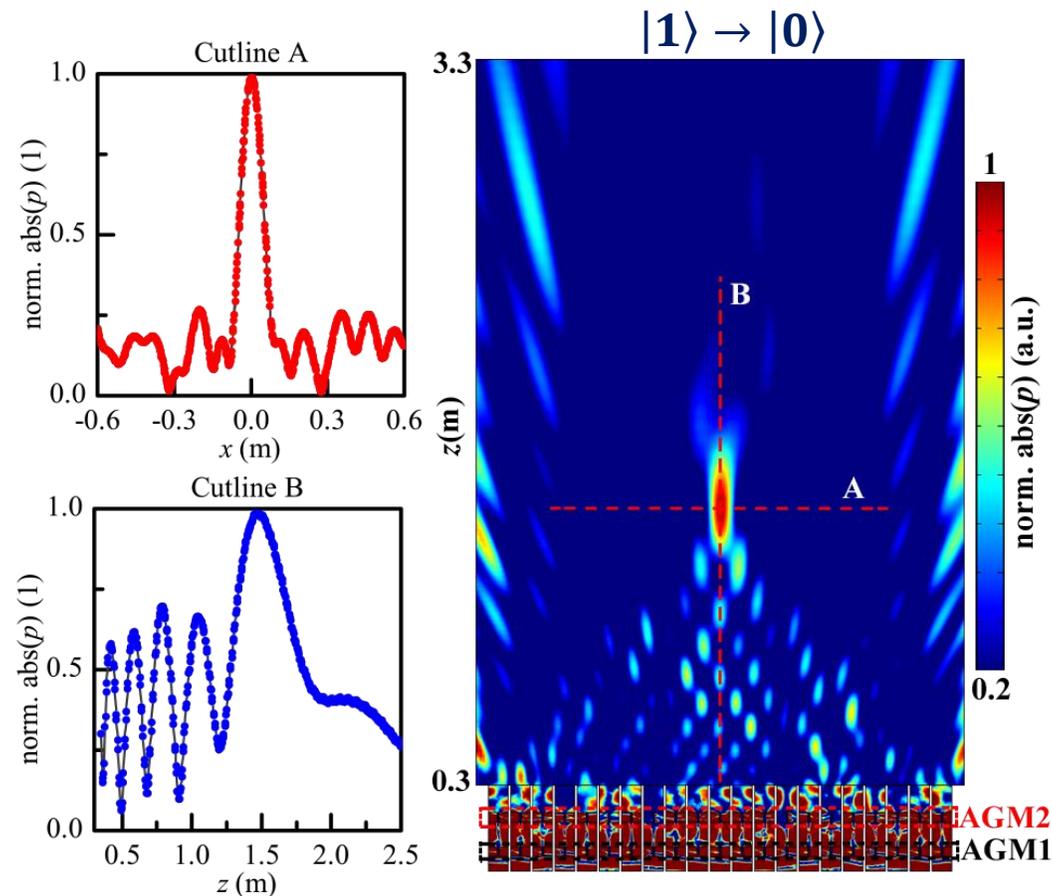
📍 波束偏折



理论:  $-6.34^\circ$   
 仿真:  $-6.3^\circ \pm 0.1^\circ$

理论:  $-12.53^\circ$   
 仿真:  $-12.8^\circ \pm 0.1^\circ$

📍 聚焦: 声学几何相位透镜



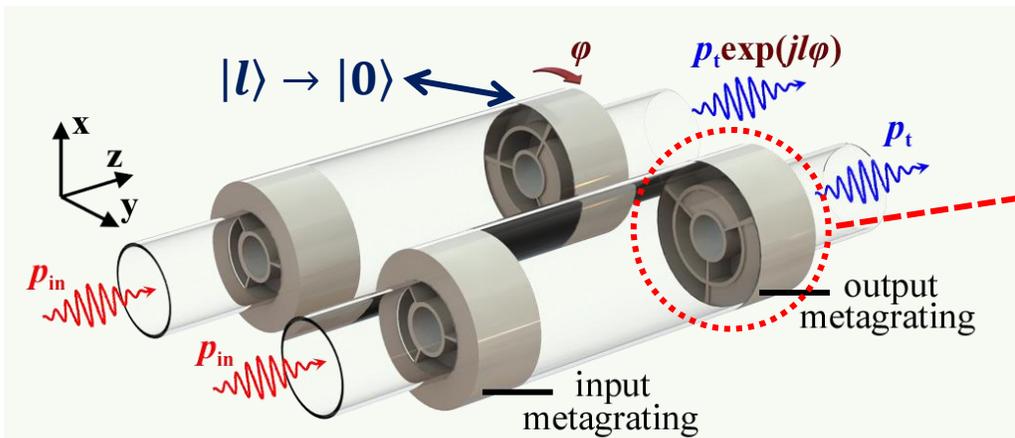
Bingyi Liu, et al. *New Journal of Physics*, 2021, 23(11), 113026.



### 3. 声学几何相位型超构表面



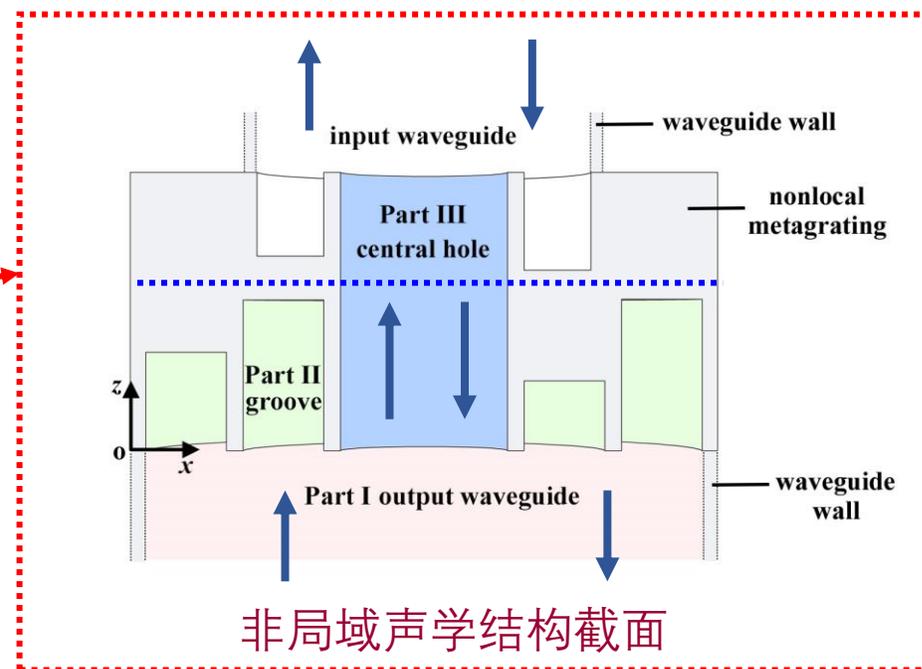
- 验证声学几何相位的一个简单方案：非局域声学结构
- 与 同济大学李勇教授团队合作



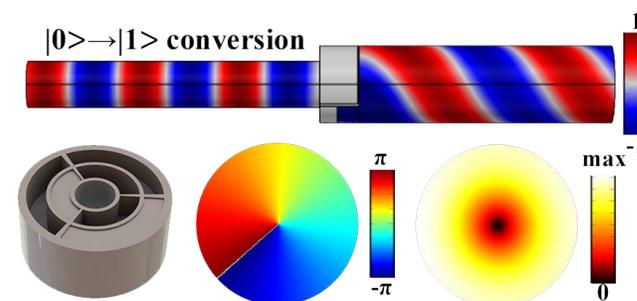
非局域结构的优点:

1. 平面波与声学涡旋的直接转换：提升效率
2. 任意阶次的几何相位均可利用两个非局域结构实现：精简结构
3. 结构间隔只需约2个波长即可：减小体积

Bingyi Liu, et al. *Applied Physics Letters*, 2022, 120(21), 211702



非局域声学结构  
声学涡旋生成



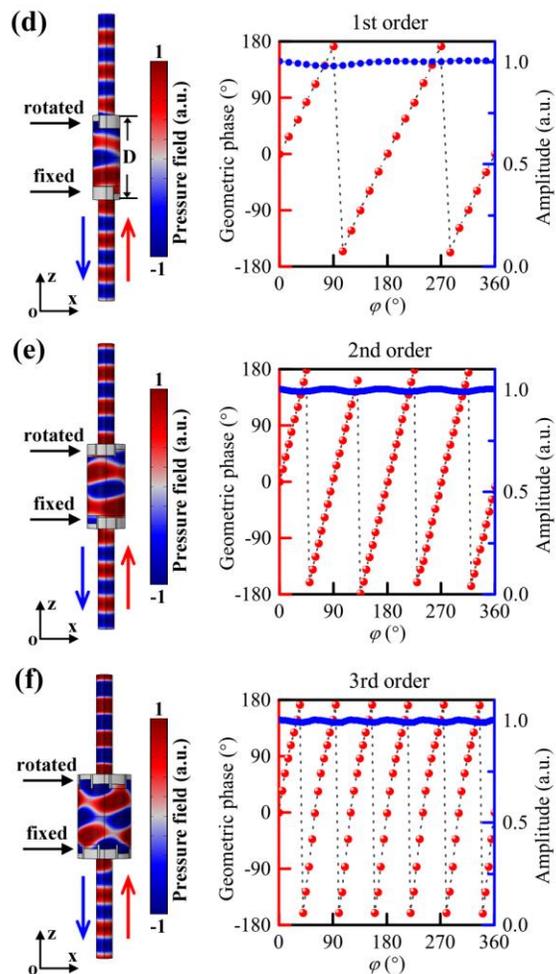
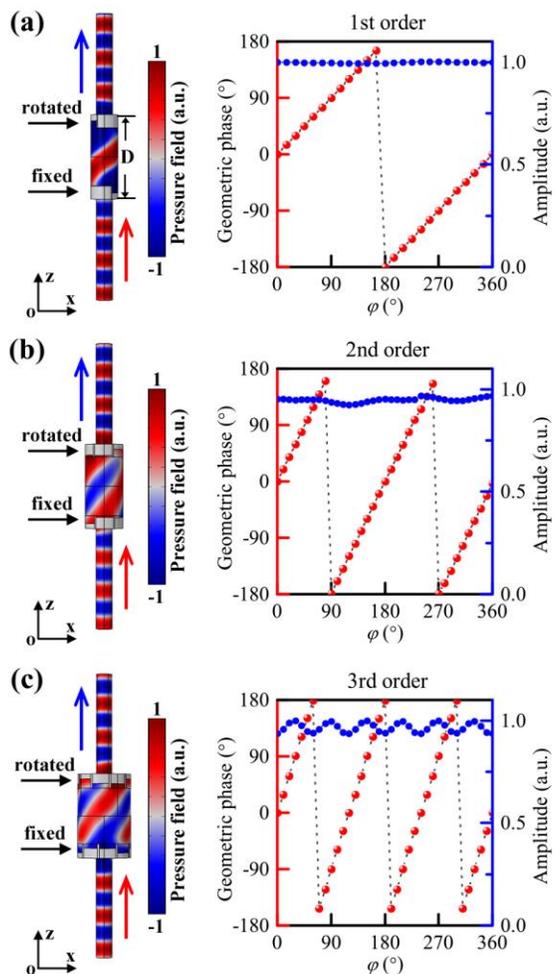


# 3. 声学几何相位型超构表面



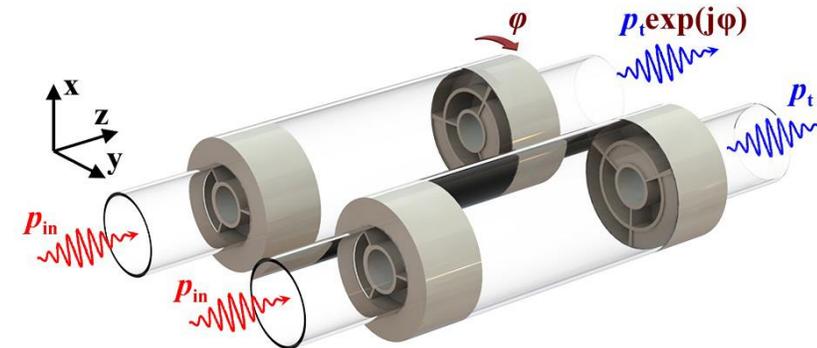
透射式

反射式

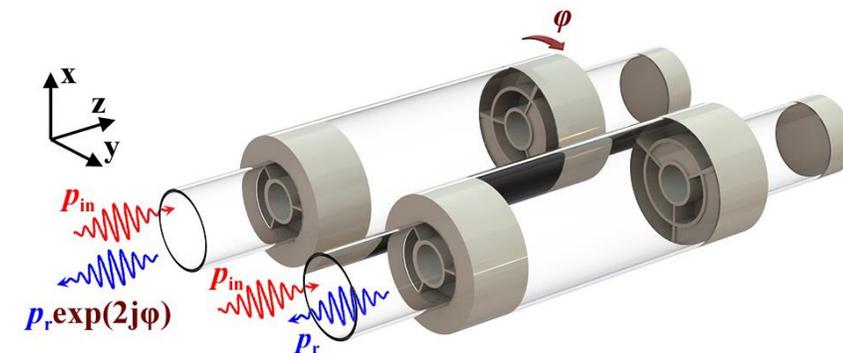


- 首次实现反射式声学几何相位
- 透射式和反射式几何相位之间满足2倍关系

Transmissive acoustic geometric phase



Reflective acoustic geometric phase

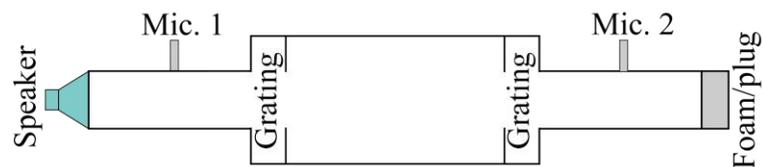




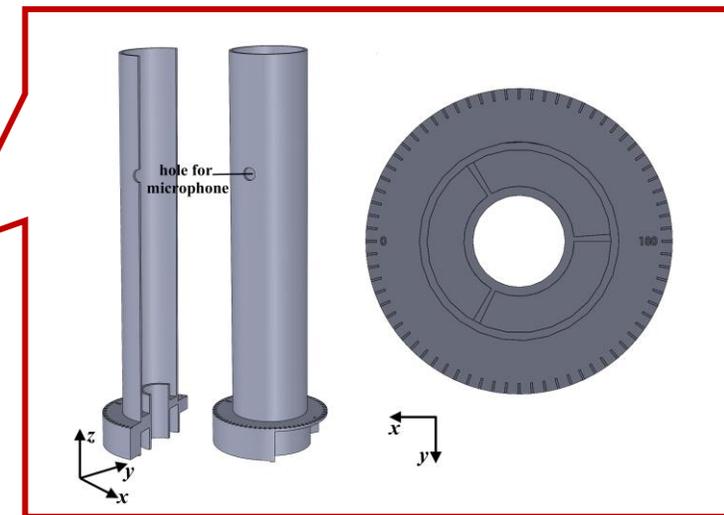
### 3. 声学几何相位型超构表面



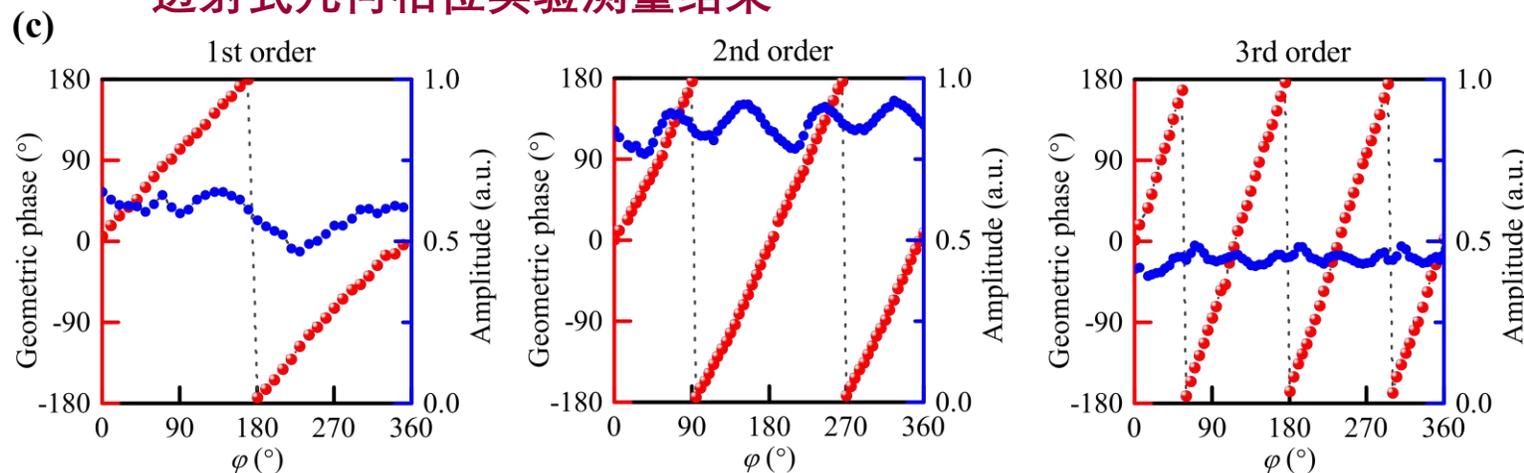
(a) 实验测量设置



(b) 样品照片



透射式几何相位实验测量结果



✓ 声波导和声学非局域结构均采用3D打印方法一体化制备

成功验证了声学几何相位!

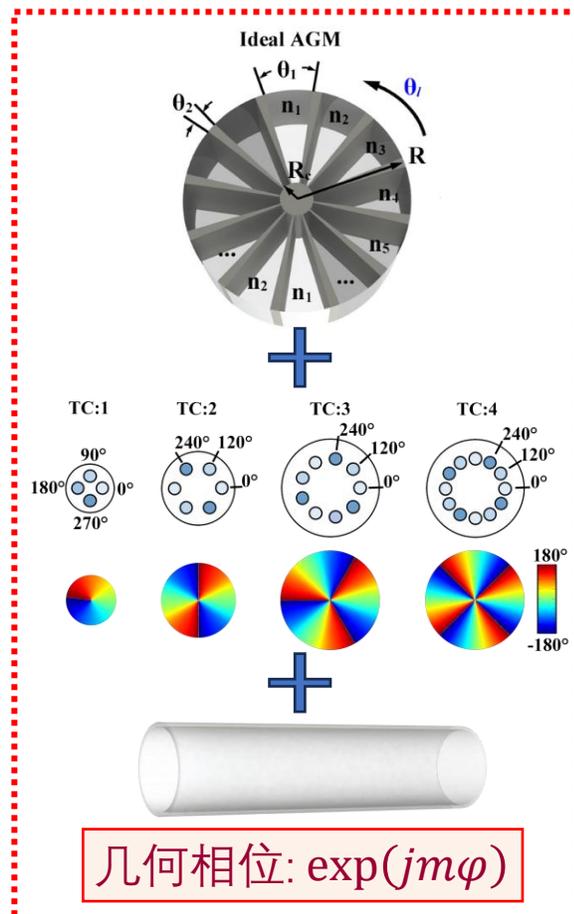
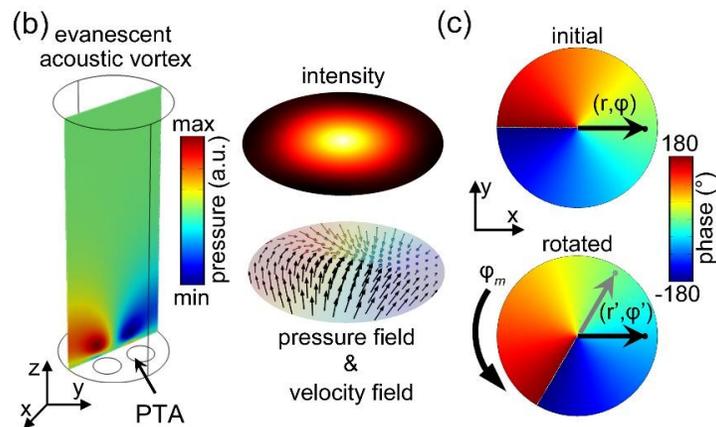
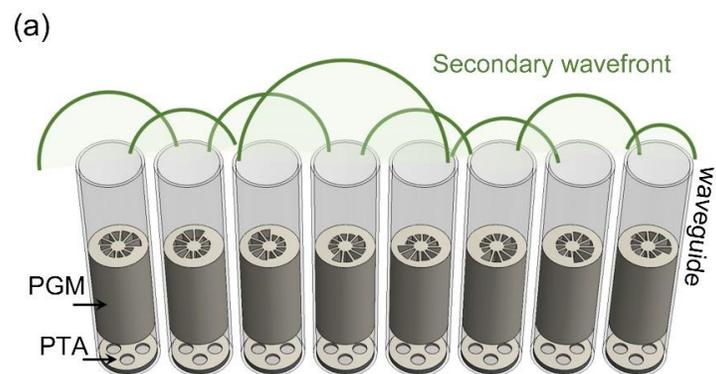
Bingyi Liu, et al. *Applied Physics Letters*, 2022, 120(21), 211702



### 3. 声学几何相位型超构表面



#### ➤ 近场耦合声学轨道角动量转换：缩小像素单元



#### ➤ 波导管中声涡旋：

$$p(r, \varphi, z) = P_{m,n} J_m(k_{m,n} r) \exp(jm\varphi) \exp(jk_z z)$$

#### ➤ 传播常数：

$$\beta_{m,n} = \frac{k_z}{k_0} = \sqrt{1 - \left(\frac{\alpha_{m,n}}{k_0 R}\right)^2}, \quad k_z = \sqrt{k_0^2 - k_{m,n}^2}$$

➤ 当  $\alpha_{m,n} > k_0 R$ ,  $\beta_{m,n}$  为纯虚数。

#### ✓ 倏逝声学涡旋：

$$p_m(r, \varphi, z) = J_m(k_{m,n} r) \exp(jm\varphi) \exp(-|k_z|z)$$

倏逝声学涡旋与梯度超构表面相互作用：

$$|0\rangle = \hat{T}(\theta_i) |m\rangle \exp(-|k_z|d)$$

将体系旋转  $\varphi_m$ ,  $r' = r$ ,  $\varphi' = \varphi - \varphi_m$ ,  $d' = d$

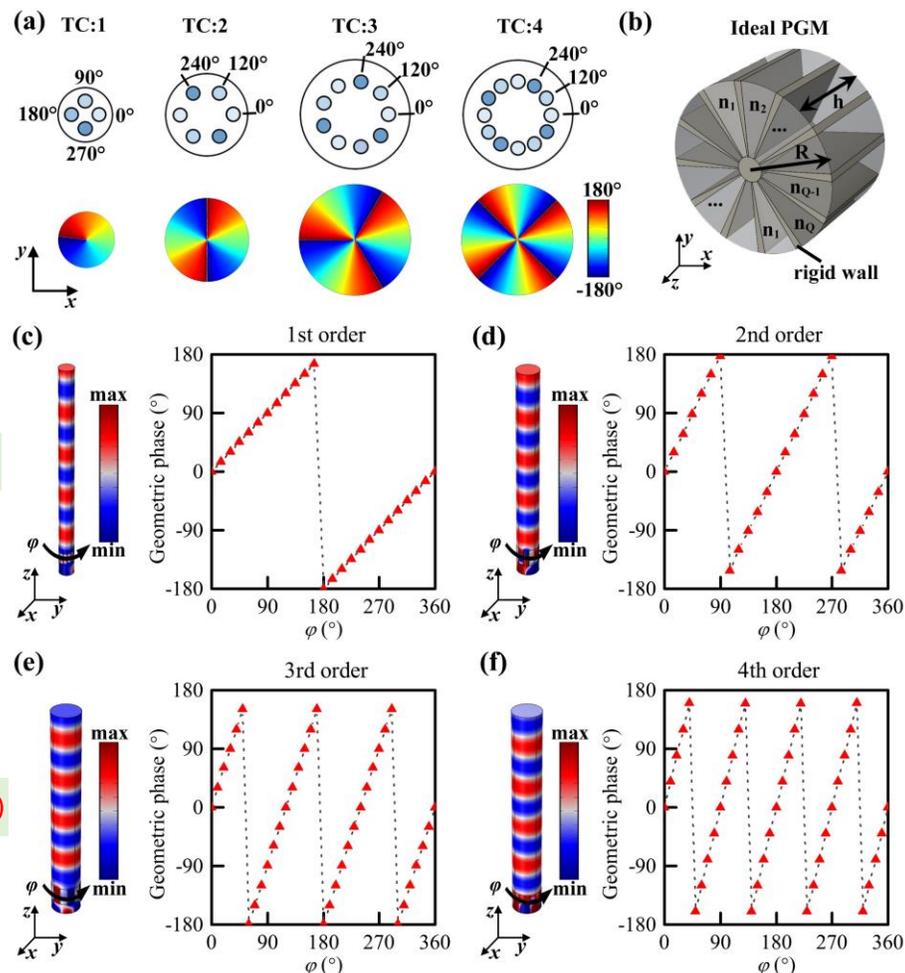
$$|0\rangle = \exp(-jm\varphi_m) \hat{T}(\theta_i + \varphi_m) |m\rangle \exp(-|k_z|d)$$

$$\hat{T}(\theta_i + \varphi_m) = \exp(jm\varphi_m) \hat{T}(\theta_i)$$

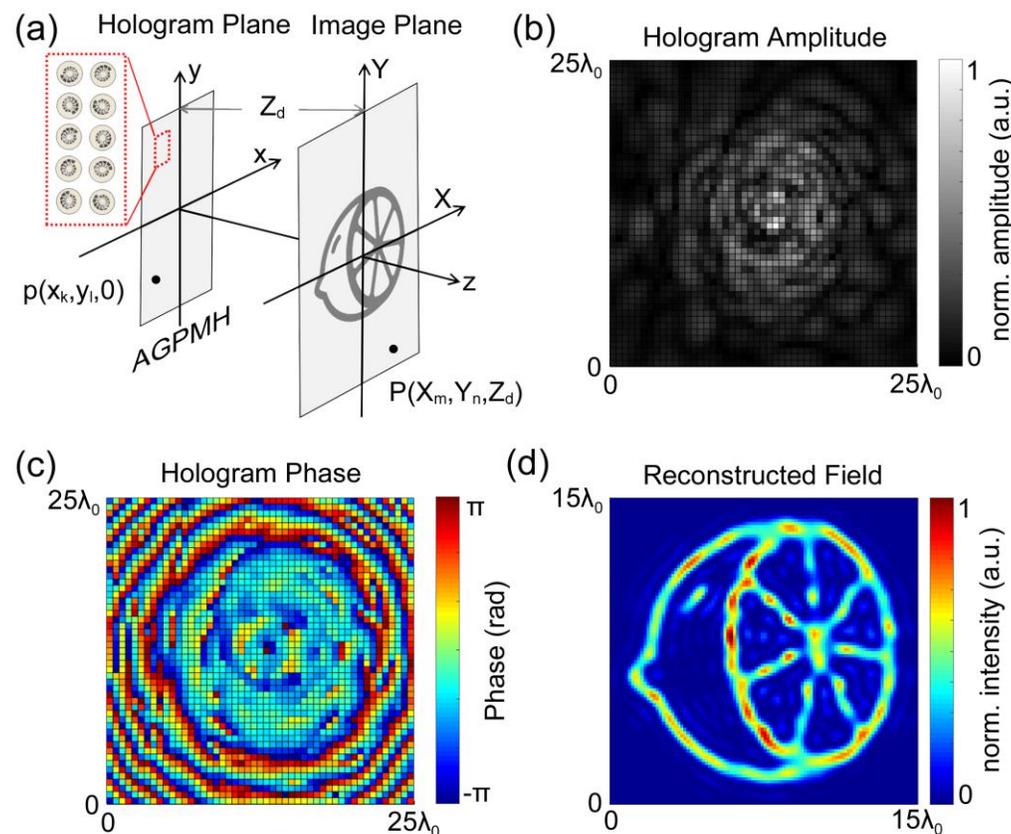
Bingyi Liu, et al. *Journal of Applied Physics*, 2022, 131(18), 185108.



# 3. 声学几何相位型超构表面



## 几何相位型声学全息



➤ 可编程+声场重构

✓ 声学几何相位:  $\exp(jm\varphi)$

Bingyi Liu, et al. *Journal of Applied Physics*, 2022, 131(18), 185108.



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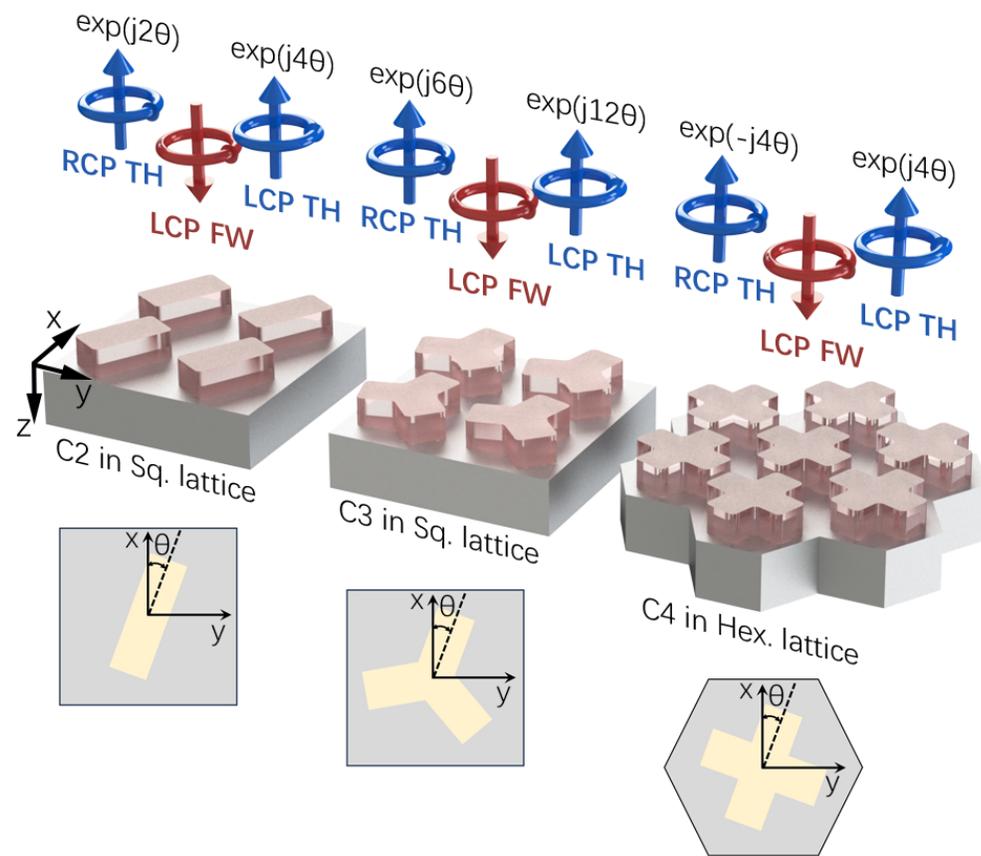
1. 研究背景
2. 非线性几何相位超表面：广义非线性几何相位
3. 声学几何相位型超构表面
- 4. 总结**



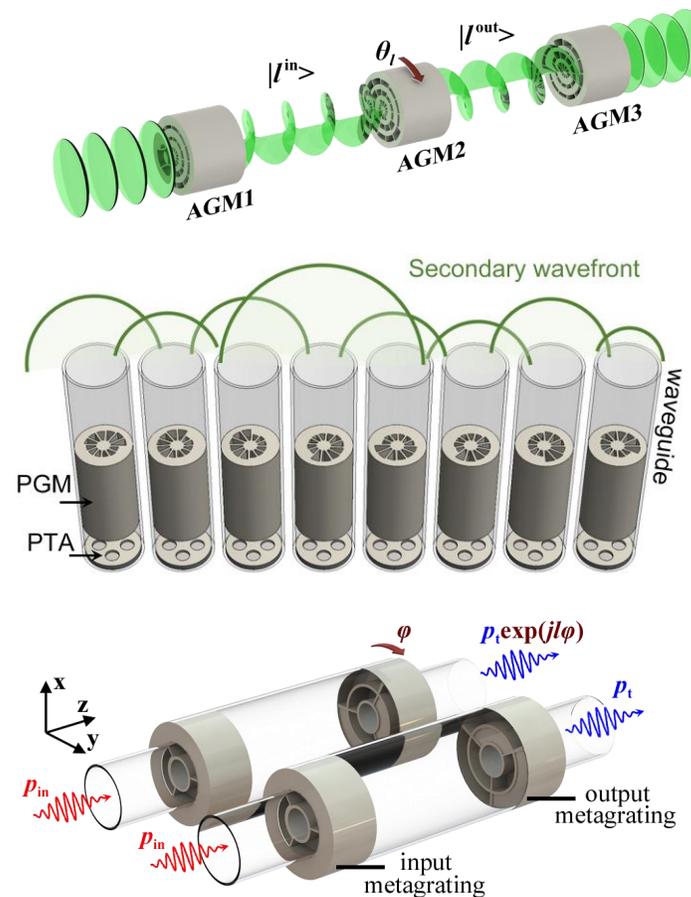
# 4. 总结



## 广义非线性几何相位



## 声学几何相位





安徽理工大学

ANHUI UNIVERSITY OF SCIENCE & TECHNOLOGY



合肥工业大学

HEFEI UNIVERSITY OF TECHNOLOGY

计算机与信息学院

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人工智能学院

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# 谢谢!



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